Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
NQSTI National Quantum Science and Technology Institute

Advancing Kinetic Inductance Traveling Wave Parametric Amplifiers for Quantum Technologies

Secondo Congresso Nazionale NQSTI February 5–7, 2025 I Rome, Italy

Pietro Campana

University of Milano-Bicocca
INFN – Milano-Bicocca
Bicocca Quantum Technologies (BiQuTe) Centre

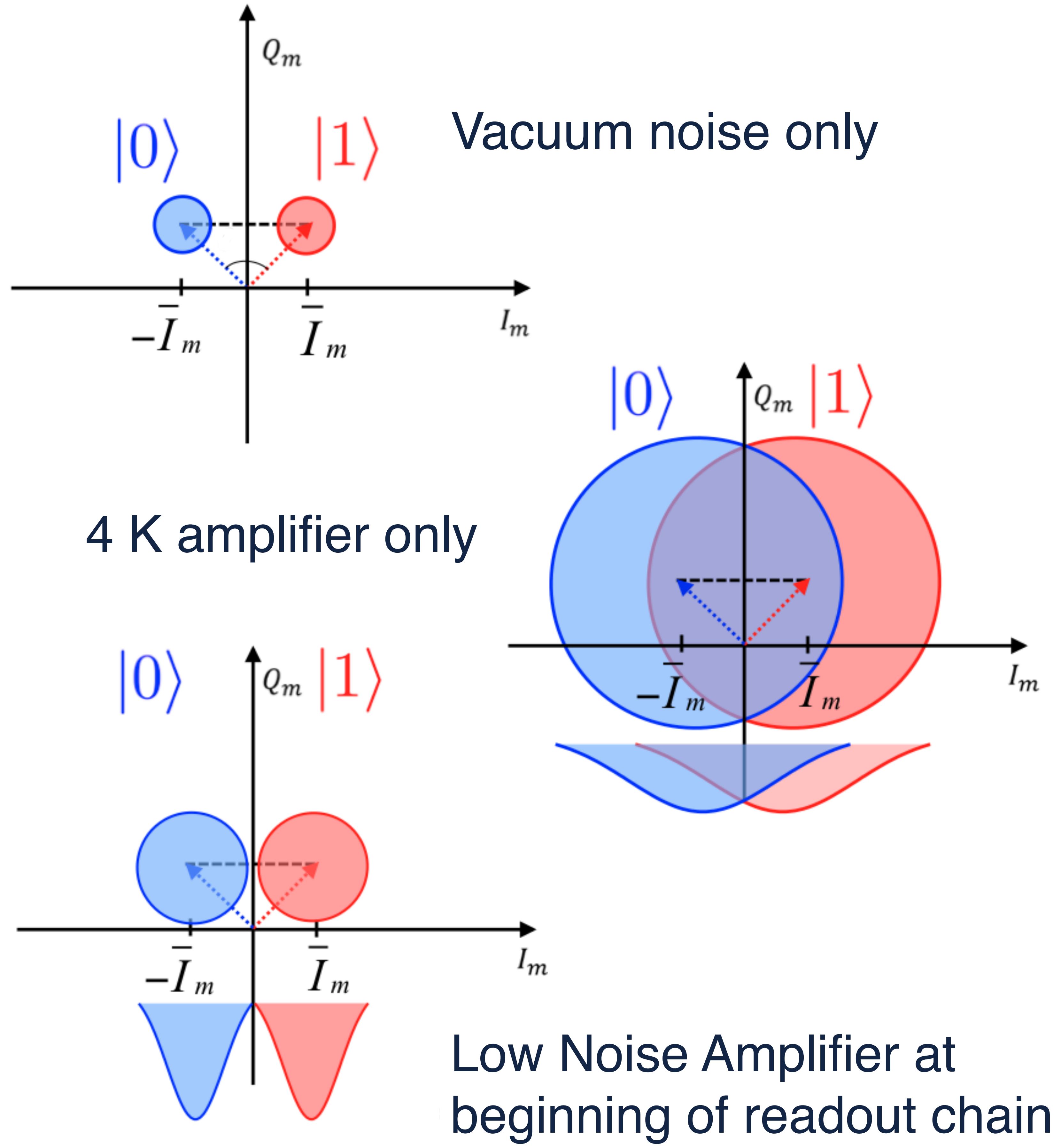
Università degli Studi di Milano Bicocca
BIQUTE
INFN Istituto Nazionale di Fisica Nucleare
DART WARS
MiSS Microwave Squeezing with Superconducting (meta) materials
ICSC Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

Finanziato dall'Unione europea
NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA
NQSTI
National Quantum Science and Technology Institute

THE QUEST FOR AN OPTIMAL AMPLIFIER

- In superconducting quantum technologies experiments, extremely weak signals need to be amplified before they are "drowned" in thermal noise.
- For high gain, the noise level of the first amplifier in the readout chain dominates the overall noise.
- As experiments become more complex, amplifiers have to satisfy additional requirements to enable multiplexing.

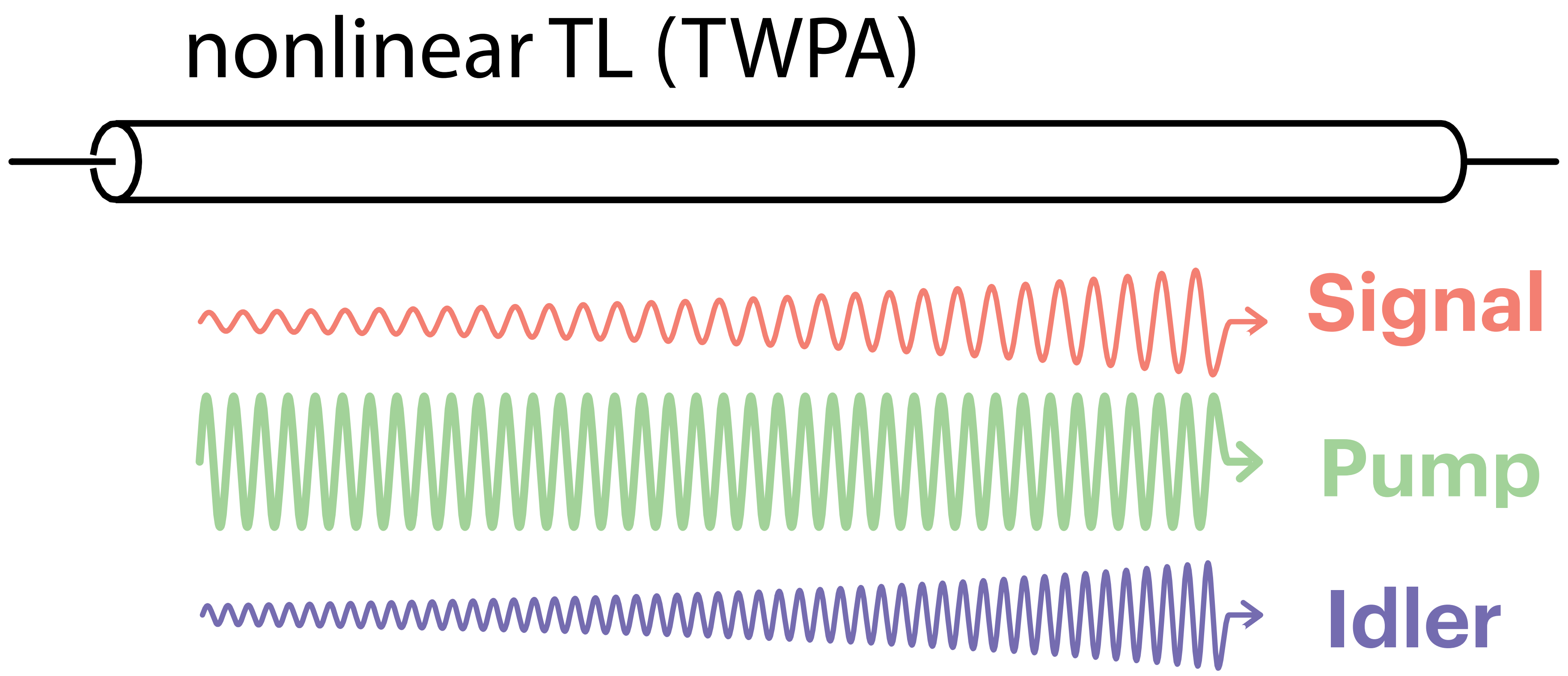
Stability	**Minimal noise**	**Gain > 20 dB**
Fabrication yield	**Multi - GHz bandwidth**	**High saturation power**



TRAVELING WAVE PARAMETERIC AMPLIFIERS

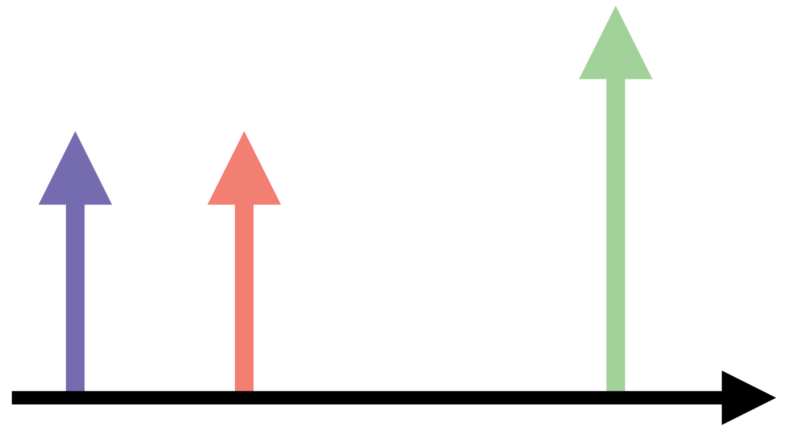
- **Standard Quantum Limit (SQL):** phase-preserving amplifiers introduce at least half a photon of noise.
- Josephson Parametric amplifiers reach the SQL, but do not satisfy other criteria for multiplexing.
- IN TWPAs, the pump, signal and idler tones exchange photons while propagating along a **nonlinear transmission line (TL).**
- Multi-GHz amplification bandwidth and high dynamic range.
- The nonlinearity usually arises from a **current-dependent inductance,** which is obtained either by using Josephson Junctions (J-TWPAs) of **Kinetic Inductance (KI-TWPAs).**

$$L(I_{DC}, I_\mu) \sim L_d(I_{DC})\left(1 + \boxed{a(I_{DC})I_\mu} + \boxed{b(I_{DC})I_\mu^2}\right)$$



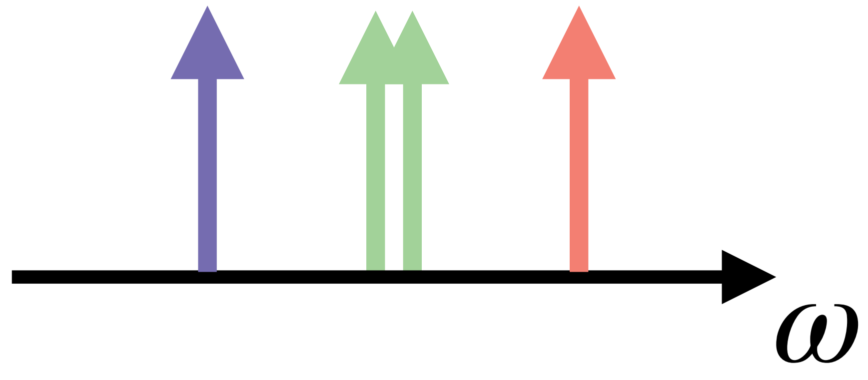
3-wave mixing:

$\omega_p = \omega_s + \omega_i$



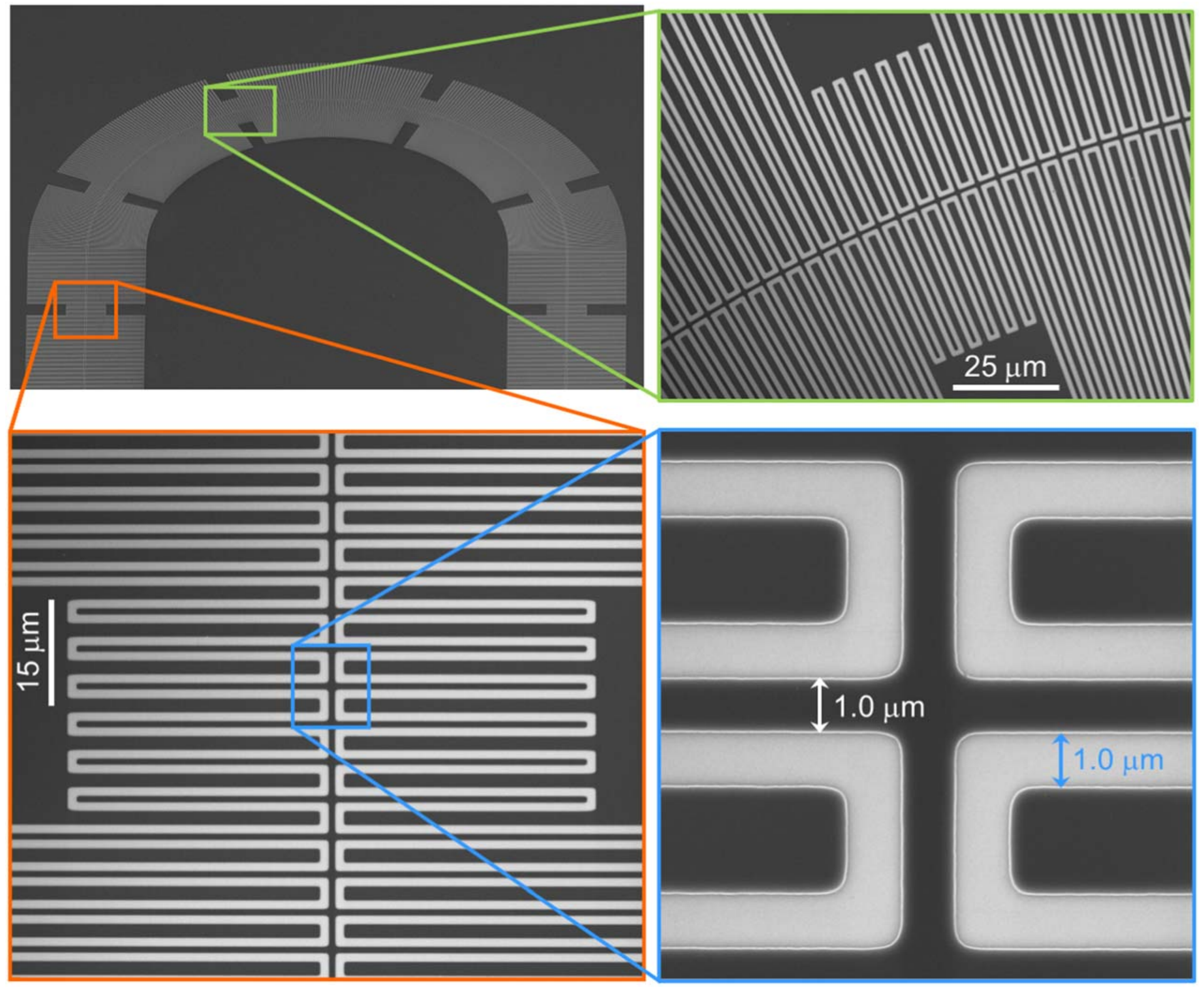
4-wave mixing:

$2\omega_p = \omega_s + \omega_i$

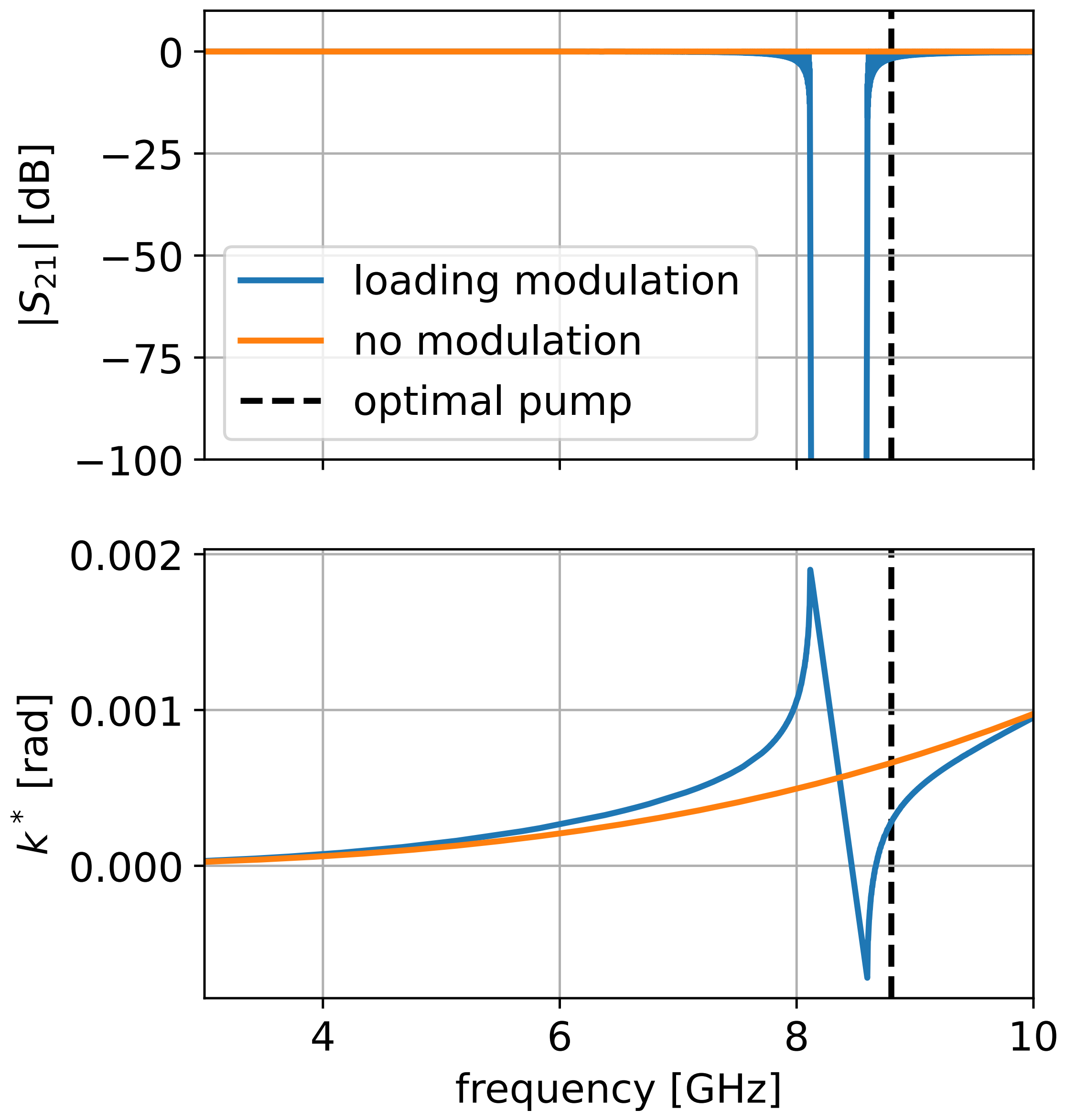


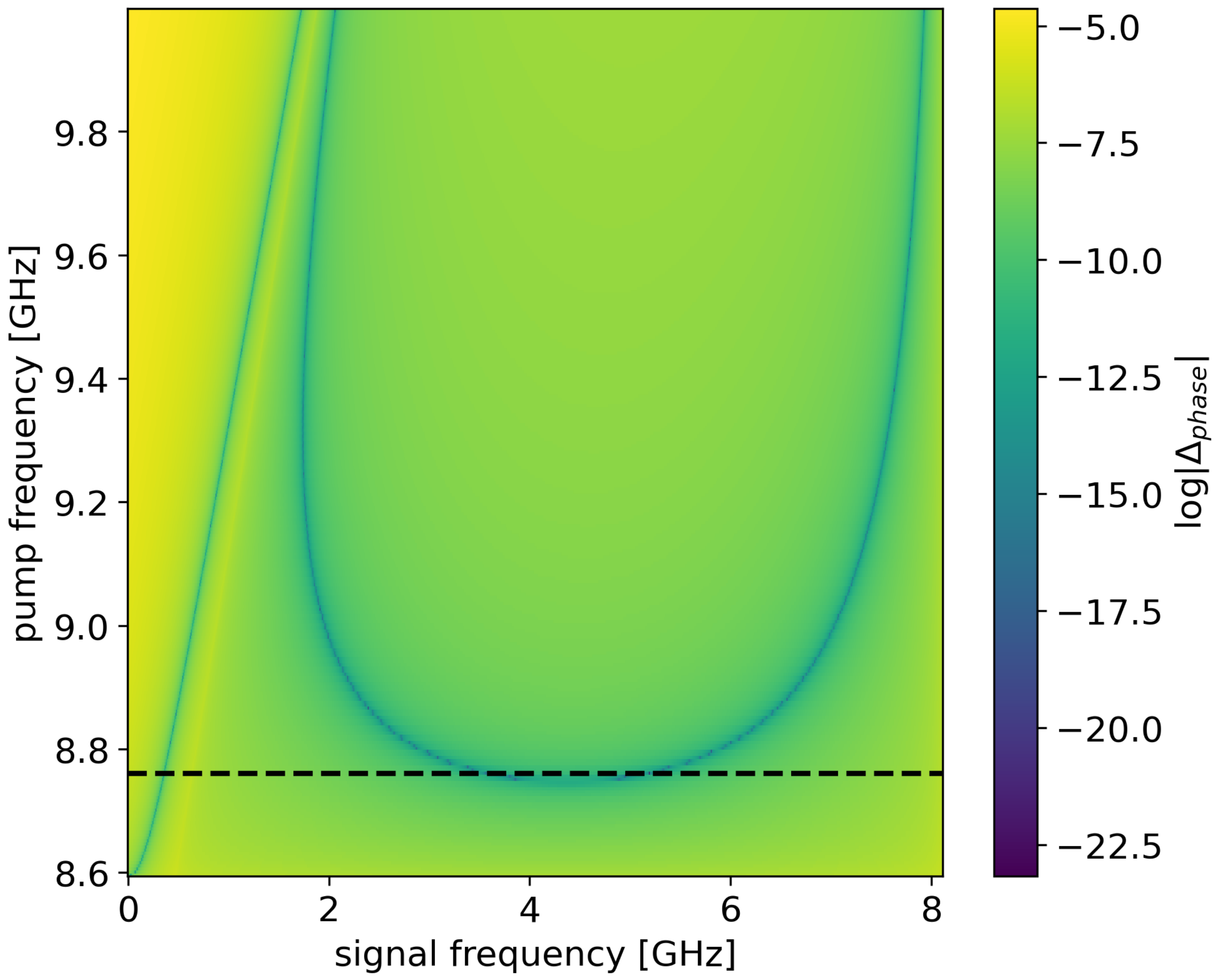
ANATOMY OF A KI-TWPA

- A transmission line is **periodically loaded with stubs or resonators** to decrease the phase velocity, **reducing its length**. Each loading section constitutes a **cell**.



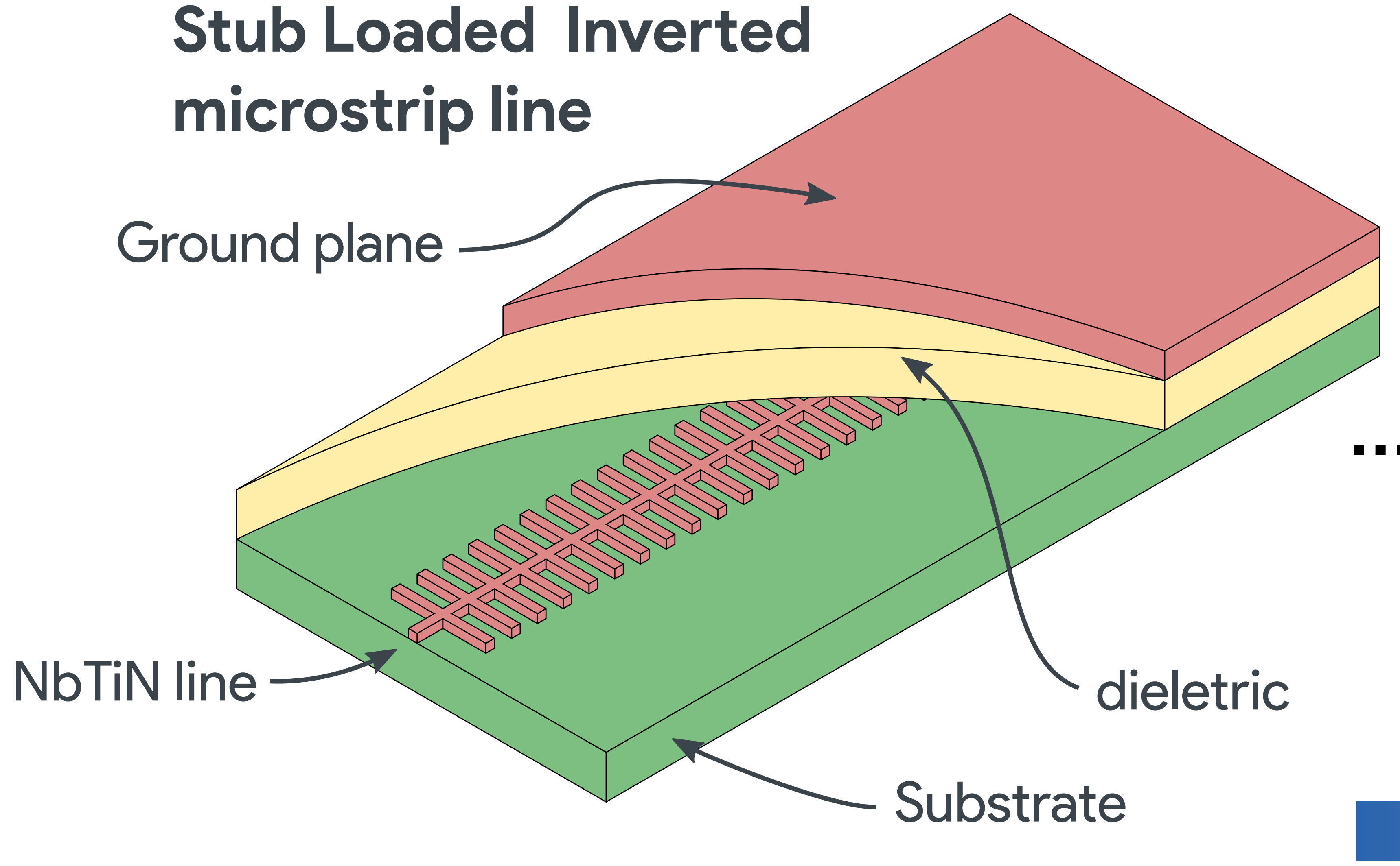
Mantegazzini, Federica, et al. Physica Scripta 98.12 (2023)

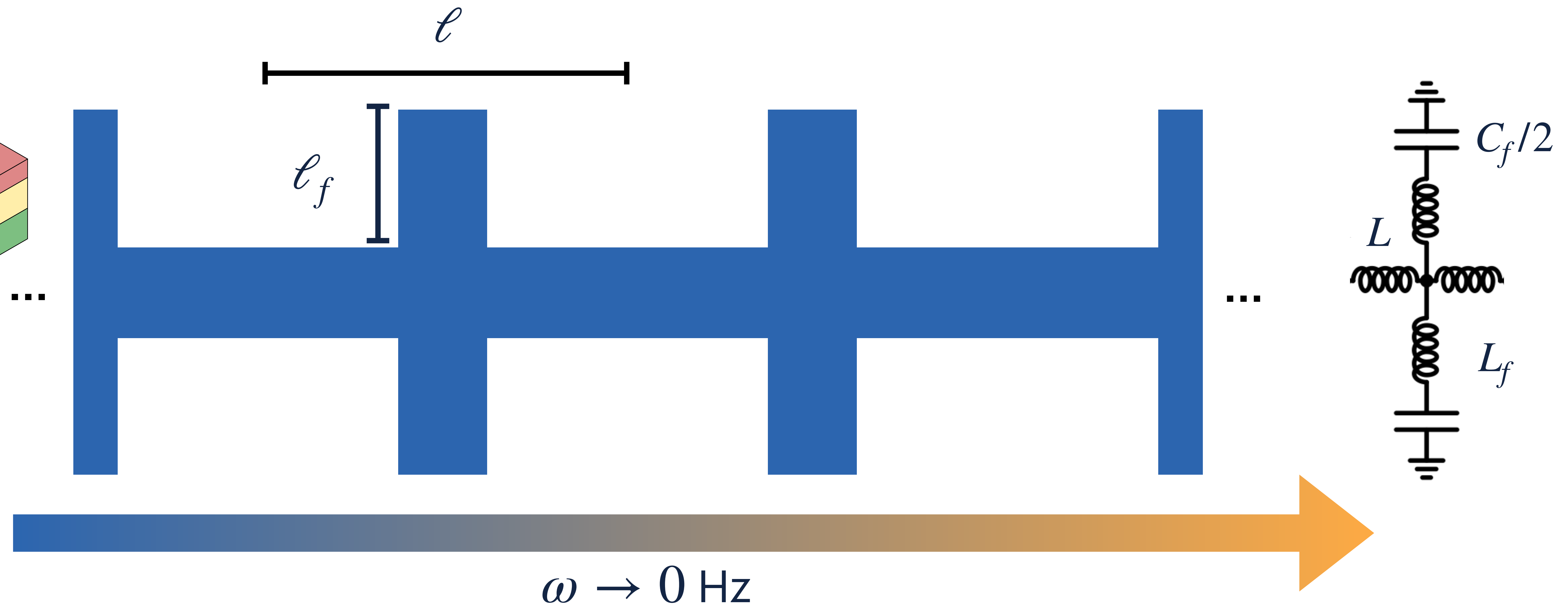




- The cells are organized in **supercells** by **modulating** the loading periodically. Most common is step modulation.
- Cell modulation controls the **dispersion relation** of the artificial TL, generating stopbands and passbands.
- Amplification occurs only when pump, signal and idler are **phase matched:** dispersion limits bandwidth to chosen range and prevents parasitic processes.

CIRCUIT MODEL OF SINGLE CELL

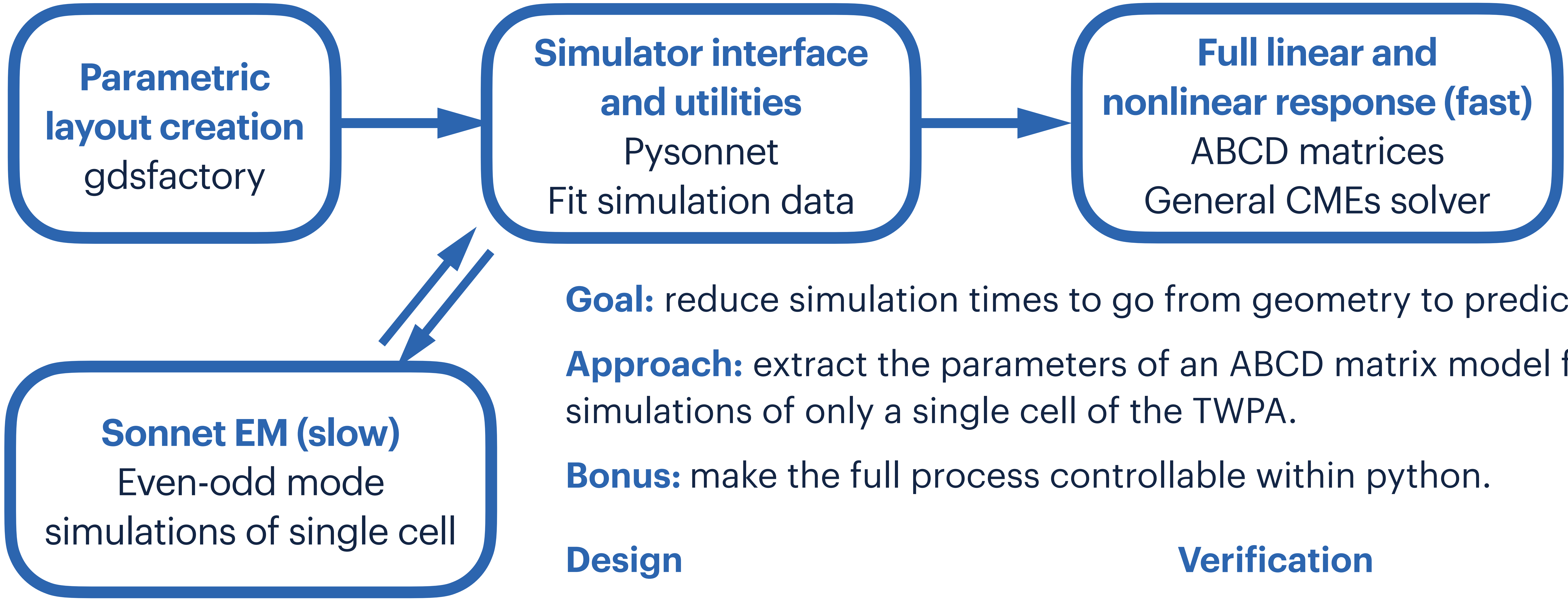




$$\begin{bmatrix}\cos(\beta\ell/2) & jZ_l\sin(\beta\ell/2)\\ j\frac{1}{Z_l}\sin(\beta\ell/2) & \cos(\beta\ell/2)\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{2}{jZ_f\cot(\beta\ell_f)} & 1\end{bmatrix}\begin{bmatrix}\cos(\beta\ell/2) & jZ_l\sin(\beta\ell/2)\\ j\frac{1}{Z_l}\sin(\beta\ell/2) & \cos(\beta\ell/2)\end{bmatrix}$$

$$\begin{bmatrix}1 & jL_d\omega/2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{j2C\omega}{2-L_fC\omega^2} & 1\end{bmatrix}\begin{bmatrix}1 & jL_d\omega/2\\ 0 & 1\end{bmatrix}$$

A FAST DESIGN AND VALIDATION FRAMEWORK FOR KI-TWPAS



Goal: reduce simulation times to go from geometry to predicted gain.

Approach: extract the parameters of an ABCD matrix model from simple EM simulations of only a single cell of the TWPA.

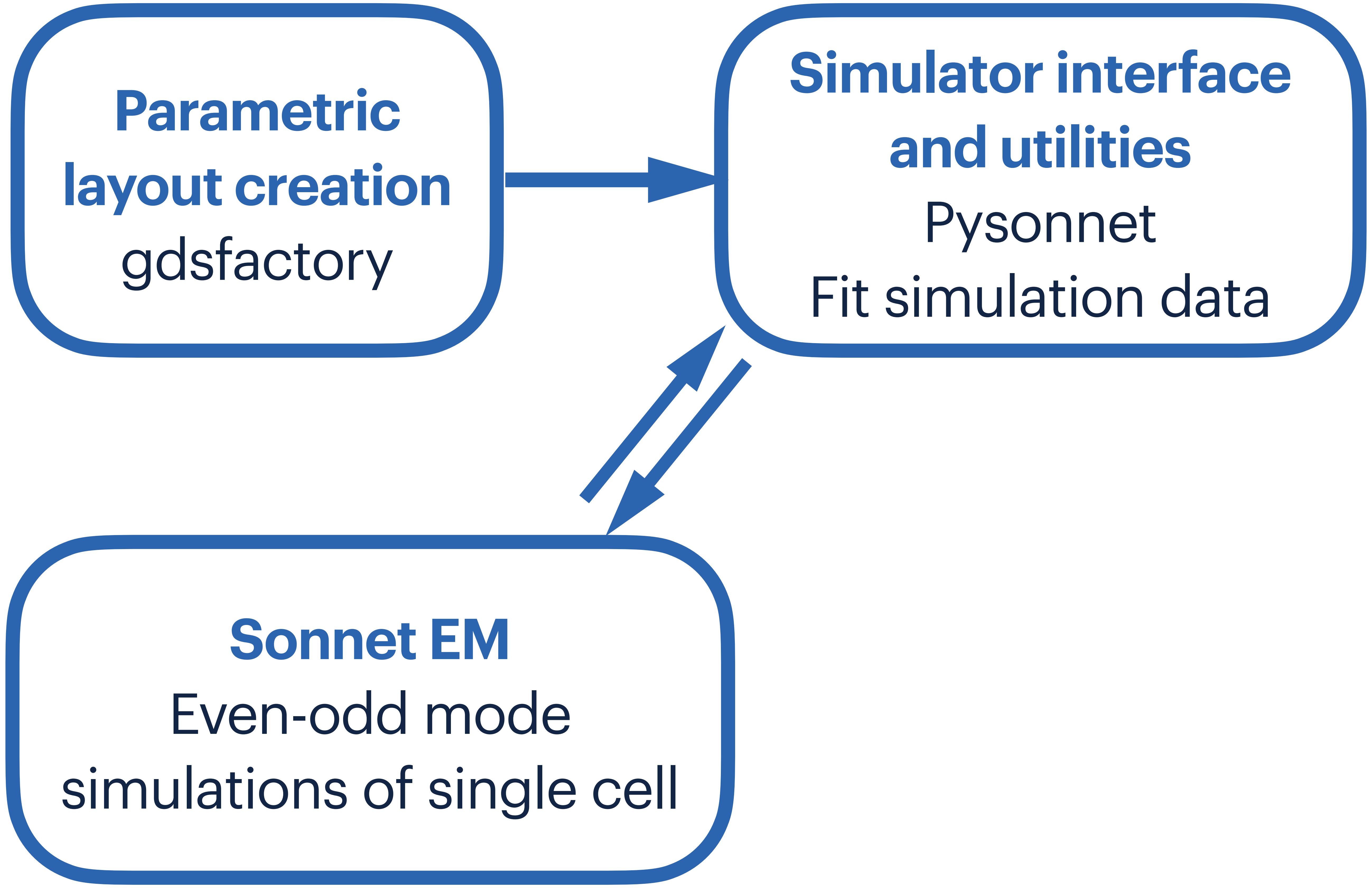
Bonus: make the full process controllable within python.

Design

- Faster synthesis of prototypes
- Numerical optimization of design parameters (eg $Z_0 = 50\ \Omega$)

Verification

- Improve correspondence of designs and measurements
- Reconstruct possible cause of deviation from expectations (eg overetching, different ϵ_r or L_k)


Parametric
layout creation
gdsfactory
Simulator interface
and utilities
Pysonnet
Fit simulation data
Sonnet EM
Even-odd mode
simulations of single cell

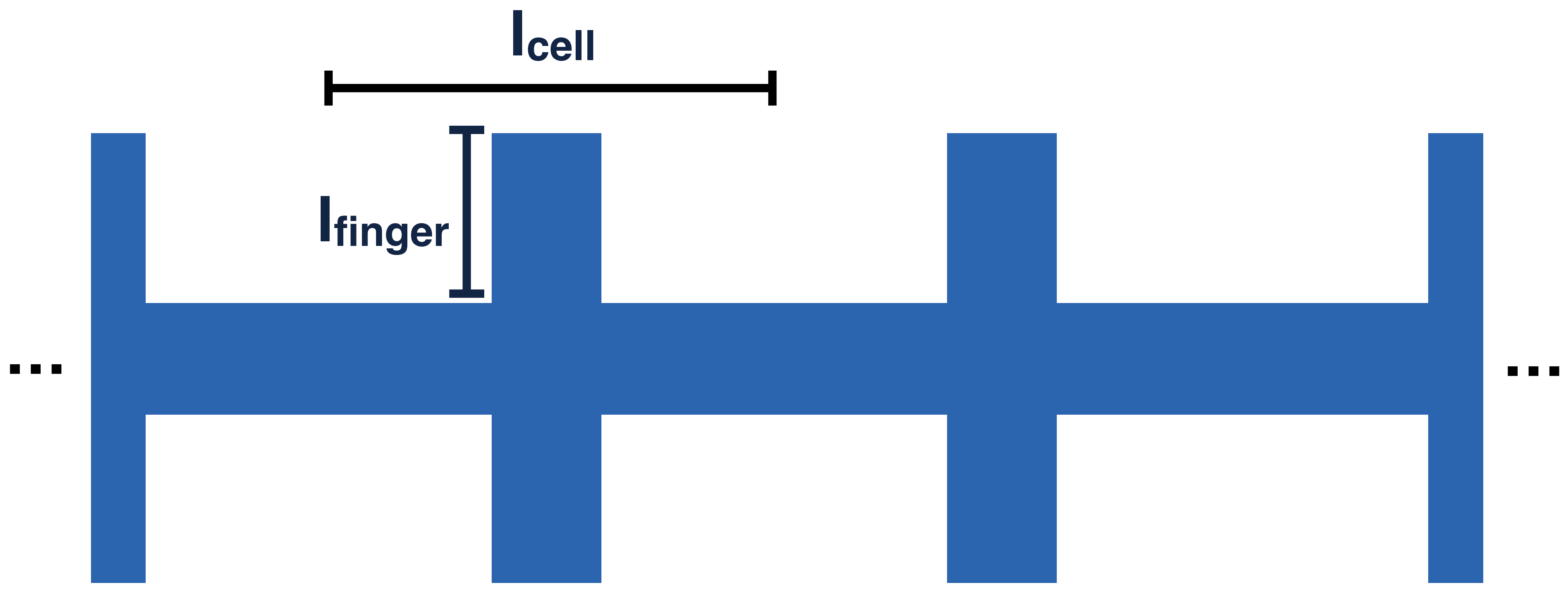
Finanziato dall'Unione europea NextGenerationEU

Ministero dell'Università e della Ricerca

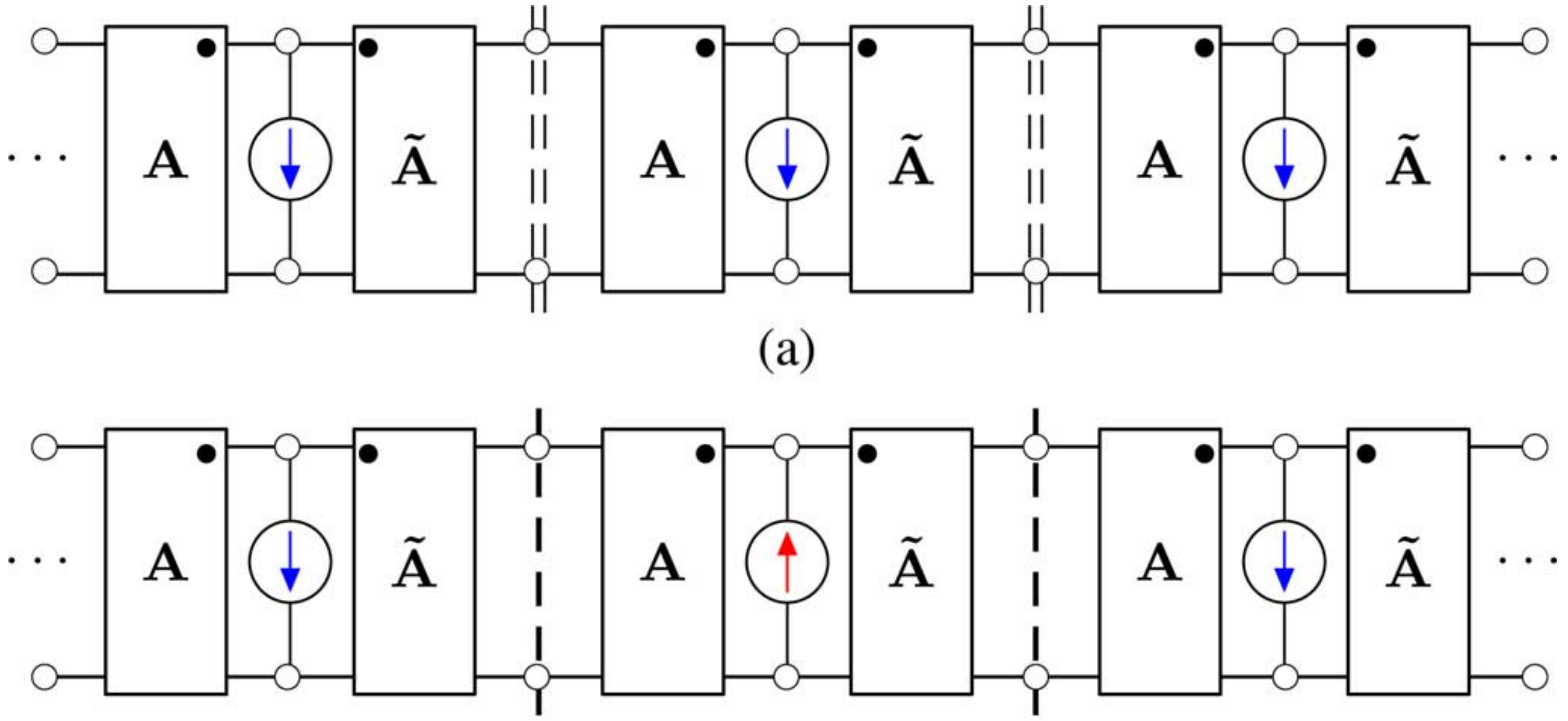
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

NQSTI National Quantum Science and Technology Institute

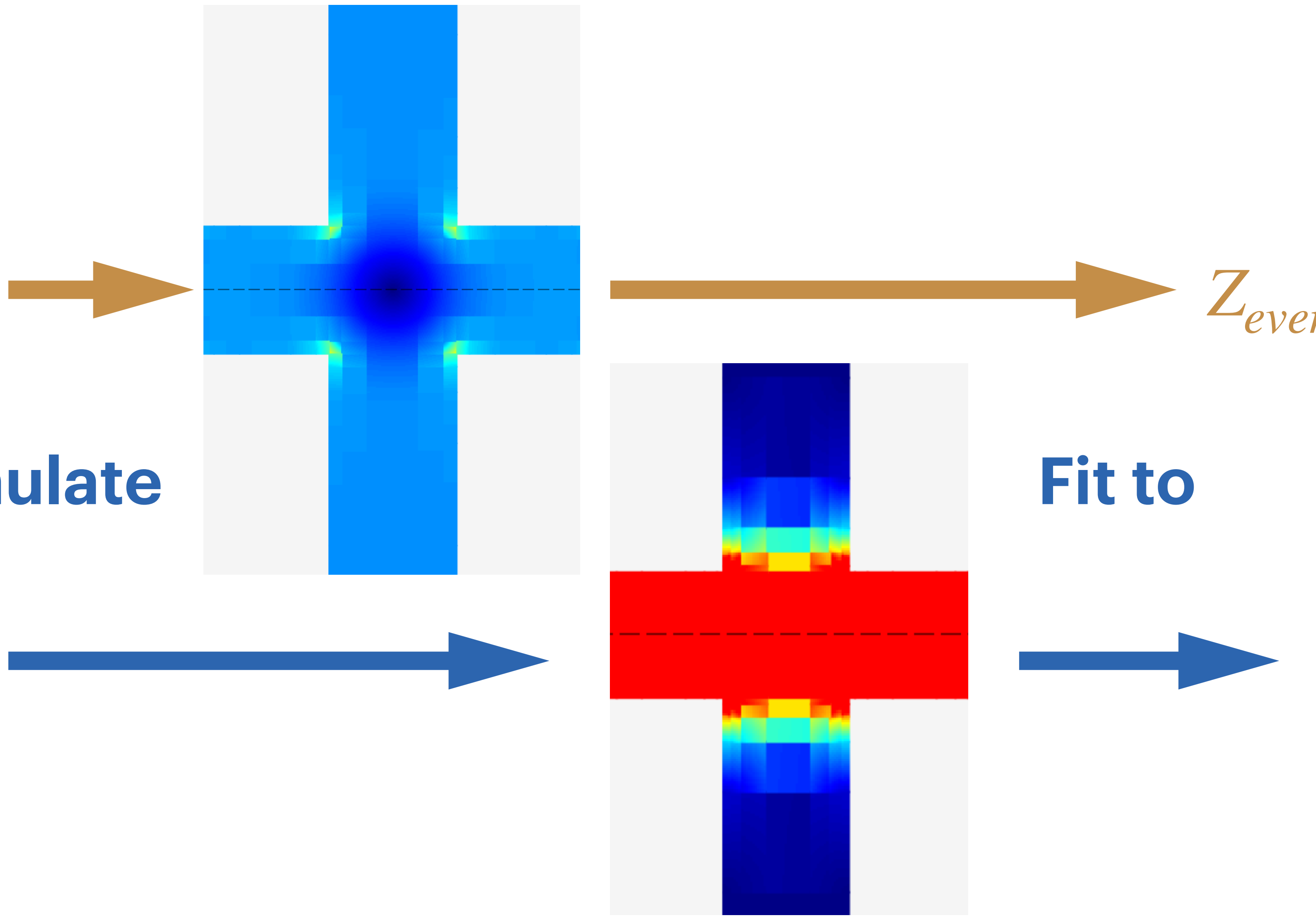
EM SIMULATIONS WITH EVEN-ODD ANALYSIS



- Need response of a cell **embedded in the periodic structure** of the TWPA.
- **Standard approach:** simulate a large number of cells, very slow.
- **New approach:** separate in two simulations of a single cell, one for the **odd** and one for the **even** mode. Use pysonnet and gdsfactory to script the simulations and their geometry within python.
- Great reduction of simulation time (more than 50x) allows to simulate with **finer meshes** and explore **larger number of configurations**.



Simulate



Fit to

$$Z_{even} = 2\left(i\sqrt{L_f/C_f}\tan(\sqrt{L_fC_f}\omega)\right)^{-1}$$

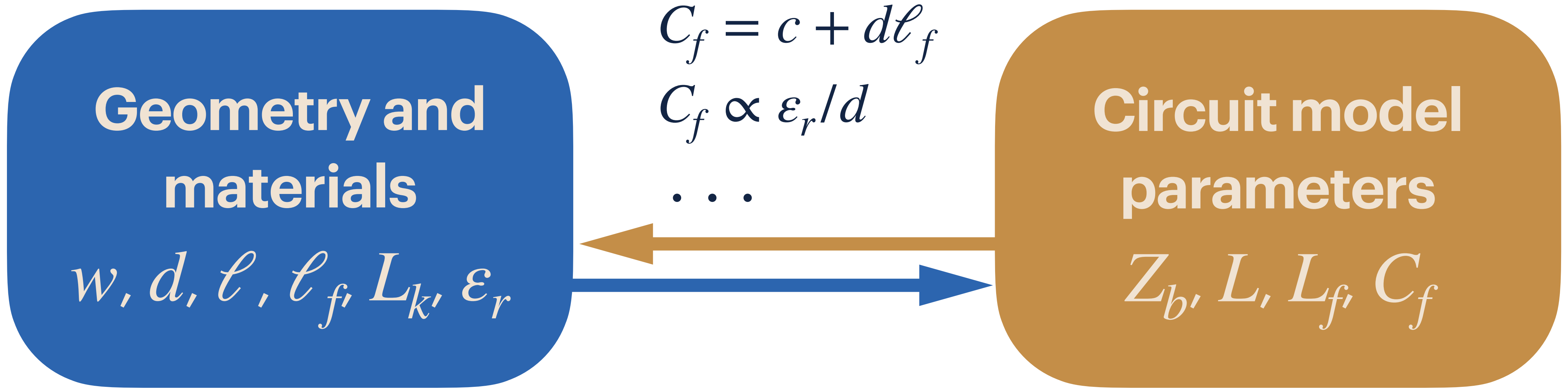
$$Z_{odd} = 2i\sqrt{L/C}\tan(\sqrt{LC}\omega/2)$$

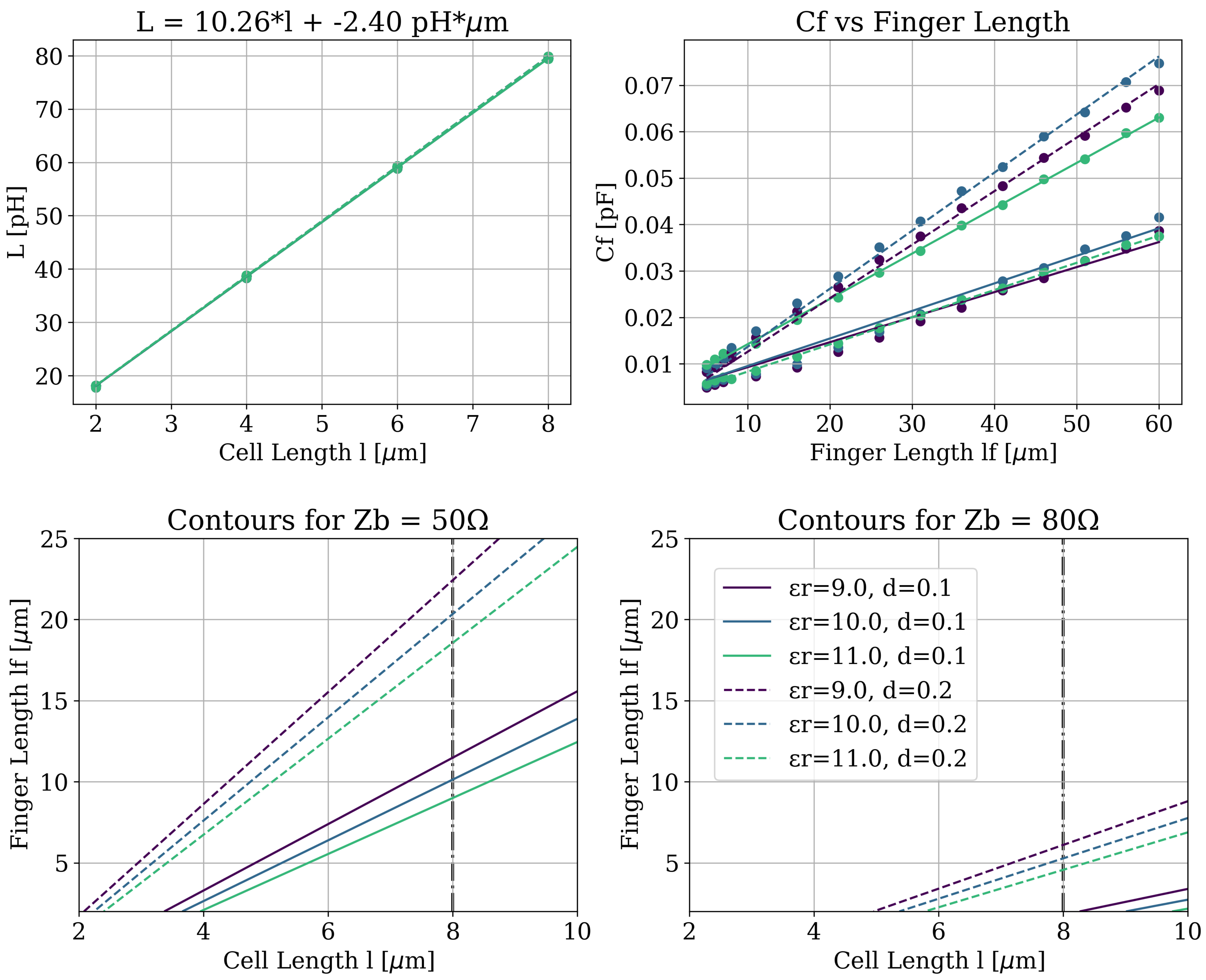
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
NQSTI National Quantum Science and Technology Institute

FROM GEOMETRY TO PARAMETERS

- Repeat the simulations and fits for different combinations of **layout geometry and materials properties.**
- This will create an equivalent set of fitted **parameters of the circuit model** of the cell.
- We can now interpolate the fitted parameters to a set of simple relations and go back and forth between the two representations.

$$L = a + b\ell$$
$$C_f = c + d\ell_f$$
$$C_f \propto \varepsilon_r / d$$
$$\dots$$





ABCD model and nonlinear response

ABCD matrices

General CMEs solver

COUPLED MODE EQUATIONS

- Insert nonlinear inductance $L = L_d\left[1 + \epsilon I + \xi I^2\right]$ in telegrapher's equations for a transmission line

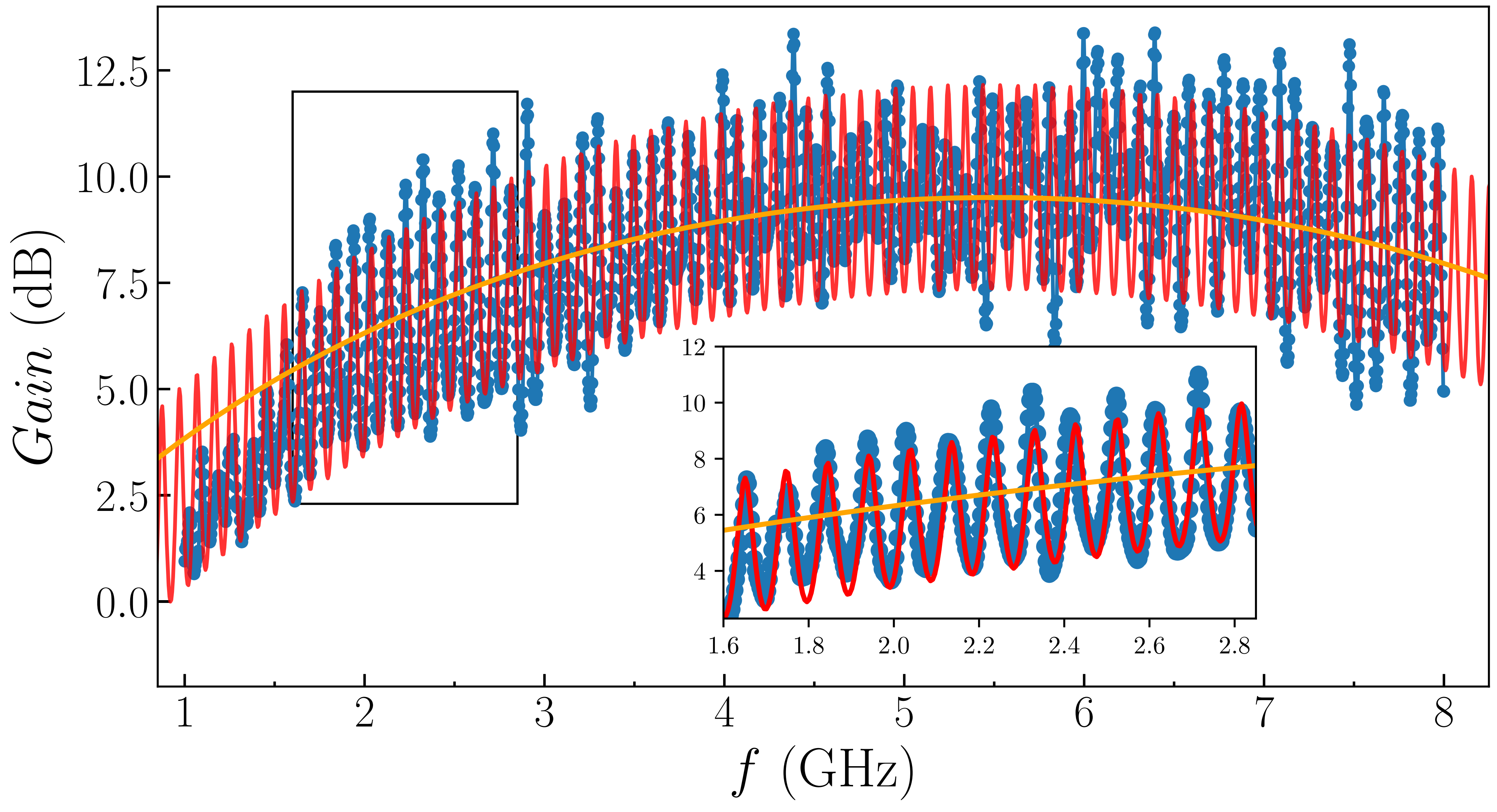
$$v_p^2 \frac{\partial^2 I}{\partial x^2} - \frac{\partial^2 I}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left(\frac{1}{2}\epsilon I^2 + \frac{1}{3}\xi I^3\right)$$

- **Ansatz:** expand current in planar waves with slow modulation along line

$$I(x,t) = \sum \left[\frac{1}{2} I_n(x) t_n \left(e^{\pm i k_n x} + \tilde{\Gamma}_n e^{\mp i k_n x}\right) e^{-i\omega_n t} + \text{c.c.}\right]$$

- Apply RWA, collect only terms which satisfy mixing processes. Result is set of **differential equations coupling input waves with different frequencies.**

$$\frac{dI_s}{dx} = i k_s(\omega) \epsilon I_p I_i^* e^{i(k(\omega)x)} + i k_s(\omega) \xi I_s \left(2|I_p|^2 + |I_s|^2 + 2|I_i|^2\right)$$



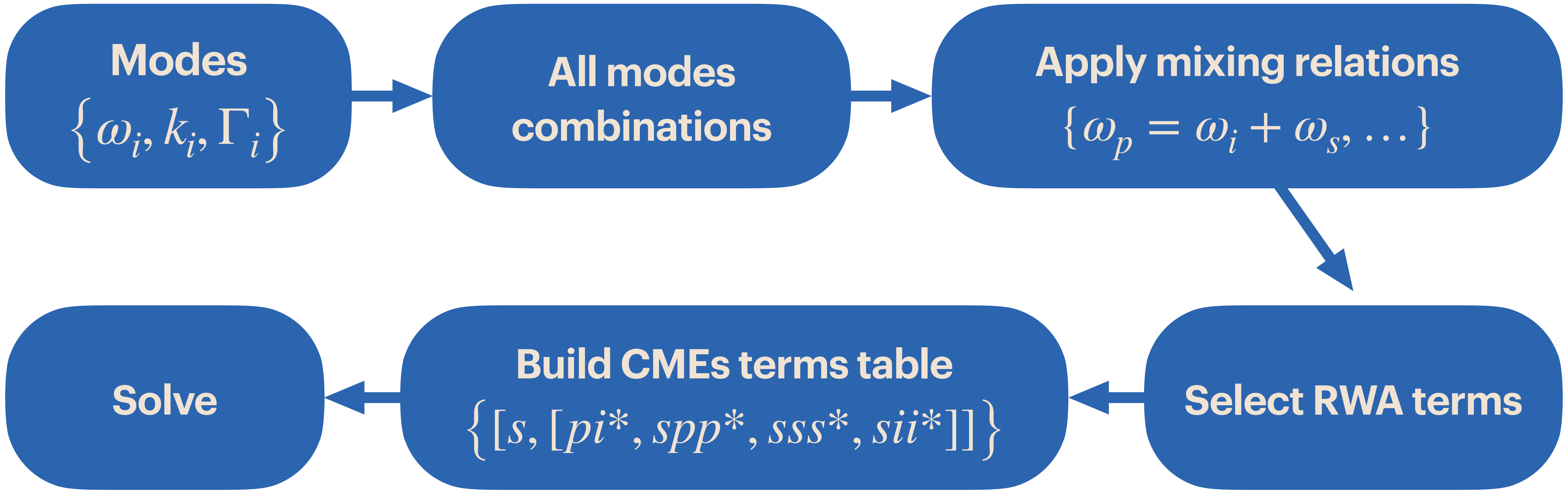
Kern, S., et al. *Physical Review B* 107.17 (2023)

TWPA PROPERTIES AND GAIN PROFILE FEATURES

- **Where does it appear?** dispersion relation $k_s(\omega)$
- **How fast does it grow with length?** Nonlinearity strength
- **Ripples:** reflections due to impedance mismatches Γ

GENERALIZED CMES SOLVER

- Additional modes and processes might be required to obtain a more realistic simulation of the gain.
- This greatly increases the complexity of the CMEs system to solve.
- Solution: create the structure of the CMEs **programmatically** before solving. Implemented using simple symbolic manipulation (sympy).



$$\frac{\partial I_a}{\partial x} = \pm i\frac{\epsilon(\omega_a)}{4}\frac{k_a}{t_a}\mathcal{F}^{si}_{\pm a}t_s t_i I_s I_i \pm i\frac{\xi(\omega_a)}{8}k_a I_a\left(\mathcal{F}^{\pm a\pm a\pm a^*}_{\pm a}|t_a|^2|I_a|^2 + 2\mathcal{F}^{\pm a\mp c\mp c^*}_{\pm a}|t_c|^2|I_c|^2 + \sum_{m\in\{s,i,d,u,c_2\}}2\mathcal{F}^{\pm amm^*}_{\pm a}|t_m|^2|I_m|^2\right) \pm i\frac{\xi(\omega_a)}{8}\frac{k_a}{t_a}\left(2\mathcal{F}^{u\mp c^* i}_{a}t_u t_c^* t_i I_u I_c^* I_i + 2\mathcal{F}^{di\mp c}_{a}t_d t_i t_c I_d I_i I_c\right) \tag{S18}$$

$$\frac{\partial I_s}{\partial x} = i\frac{\epsilon(\omega_s)}{4}\frac{k_s}{t_s}\left(\mathcal{F}^{\pm ai^*}_{s}t_a t_i^* I_a I_i^* + \mathcal{F}^{\mp cd}_{s}t_c t_d I_c I_d + \mathcal{F}^{u\mp c^*}_{s}t_u t_c^* I_u I_c^*\right) + i\frac{\xi(\omega_s)}{8}k_s I_s\left(\mathcal{F}^{sss^*}_{s}|t_s|^2|I_s|^2 + 2\mathcal{F}^{s\pm a\pm a^*}_{s}|t_a|^2|I_a|^2 + 2\mathcal{F}^{s\mp c\mp c^*}_{s}|t_c|^2|I_c|^2 + \sum_{m\in\{i,d,u,c_2\}}2\mathcal{F}^{smm^*}_{s}|t_m|^2|I_m|^2\right) + i\frac{\xi(\omega_s)}{8}\frac{k_s}{t_s}\left(2\mathcal{F}^{c_2\mp c^* d}_{s}t_{c_2}t_c^* t_d I_{c_2}I_c^* I_d + 2\mathcal{F}^{uc_2^*\mp c}_{s}t_u t_{c_2}^* t_c I_u I_{c_2}^* I_c\right) \tag{S19}$$

$$\frac{\partial I_i}{\partial x} = i\frac{\epsilon(\omega_i)}{4}\frac{k_i}{t_i}\mathcal{F}^{\pm as^*}_{i}t_a t_s^* I_a I_s^* + i\frac{\xi(\omega_i)}{8}k_i I_i\left(\mathcal{F}^{iii^*}_{i}|t_i|^2|I_i|^2 + 2\mathcal{F}^{i\pm a\pm a^*}_{i}|t_a|^2|I_a|^2 + 2\mathcal{F}^{i\mp c\mp c^*}_{i}|t_c|^2|I_c|^2 + \sum_{m\in\{s,d,u,c_2\}}2\mathcal{F}^{imm^*}_{i}|t_m|^2|I_m|^2\right) + i\frac{\xi(\omega_i)}{8}\frac{k_i}{t_i}\left(2\mathcal{F}^{\pm au^*\mp c}_{i}t_a t_u^* t_c I_a I_u^* I_c + 2\mathcal{F}^{\pm ad^*\mp c^*}_{i}t_a t_d^* t_c^* I_a I_d^* I_c^*\right) \tag{S20}$$

DO NOT WRITE EXPLICITLY

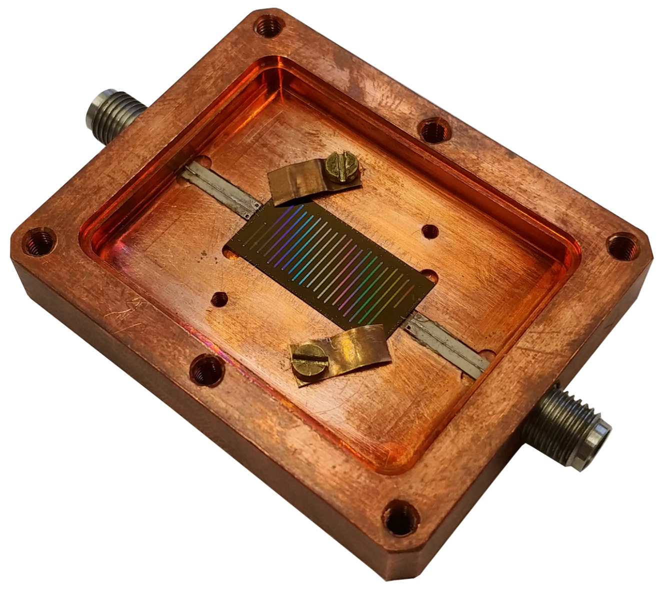
$$\frac{\partial I_c}{\partial x} = \mp i\frac{\epsilon(\omega_c)}{4}\frac{k_c}{t_c}\left(\mathcal{F}^{[illegible]}_{\mp c}[illegible]t_d^* I[illegible] + \mathcal{F}^{[illegible]}_{\mp}[illegible]t_s^* I_u[illegible] + \mathcal{F}^{[illegible]}_{[illegible]}t_{c_2}[illegible]I_{c_2}[illegible]\right) \mp i\frac{\xi(\omega_c)}{8}k_c I_c\left(\mathcal{F}^{\mp c\mp c\mp c^*}_{\mp c}|t_c|^2|I_c|^2 + 2\mathcal{F}^{\mp c\pm a\pm a^*}_{\mp c}|t_a|^2|I_a|^2 + \sum_{m\in\{s,i,d,u,c_2\}}2\mathcal{F}^{\mp cmm^*}_{\mp c}|t_m|^2|I_m|^2\right) \mp i\frac{\xi(\omega_c)}{8}\frac{k_c}{t_c}\left(2\mathcal{F}^{ud^*\mp c^*}_{\mp c}t_u t_d^* t_c^* I_u I_d^* I_c^* + 2\mathcal{F}^{c_2 s^* d}_{\mp c}t_{c_2}t_s^* t_d I_{c_2}I_s^* I_d + 2\mathcal{F}^{c_2 u^* s}_{\mp c}t_{c_2}t_u^* t_s I_{c_2}I_u^* I_s + 2\mathcal{F}^{u\pm a^* i}_{\mp c}t_u t_a^* t_i I_u I_a^* I_i + 2\mathcal{F}^{\pm ad^* i^*}_{\mp c}t_a t_d^* t_i^* I_a I_d^* I_i^*\right) \tag{S21}$$

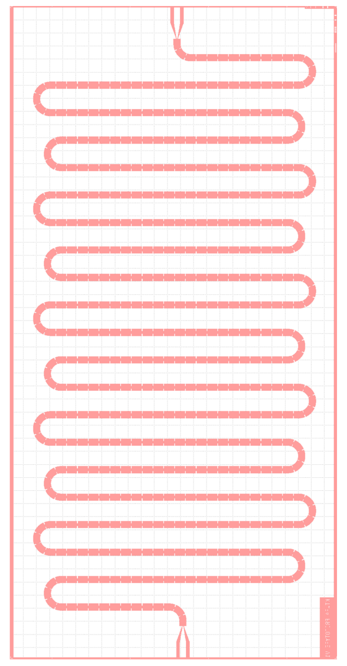
$$\frac{\partial I_d}{\partial x} = i\frac{\epsilon(\omega_d)}{4}\frac{k_d}{t_d}\left(\mathcal{F}^{s\mp c^*}_{d}t_s t_c^* I_s I_c^* + \mathcal{F}^{uc_2^*}_{d}t_u t_{c_2}^* I_u I_{c_2}^*\right) + i\frac{\xi(\omega_d)}{8}k_d I_d\left(\mathcal{F}^{ddd^*}_{d}|t_d|^2|I_d|^2 + 2\mathcal{F}^{d\pm a\pm a^*}_{d}|t_a|^2|I_a|^2 + 2\mathcal{F}^{d\mp c\mp c^*}_{d}|t_c|^2|I_c|^2 + \sum_{m\in\{s,i,u,c_2\}}2\mathcal{F}^{dmm^*}_{d}|t_m|^2|I_m|^2\right) + i\frac{\xi(\omega_d)}{8}\frac{k_d}{t_d}\left(\mathcal{F}^{u\mp c^*\mp c^*}_{d}t_u t_c^{*2}I_u I_c^{*2} + 2\mathcal{F}^{\mp csc_2^*}_{d}t_c t_s t_{c_2}^* I_c I_s I_{c_2}^* + 2\mathcal{F}^{\pm ai^*\mp c^*}_{d}t_a t_i^* t_c^* I_a I_i^* I_c^*\right) \tag{S22}$$

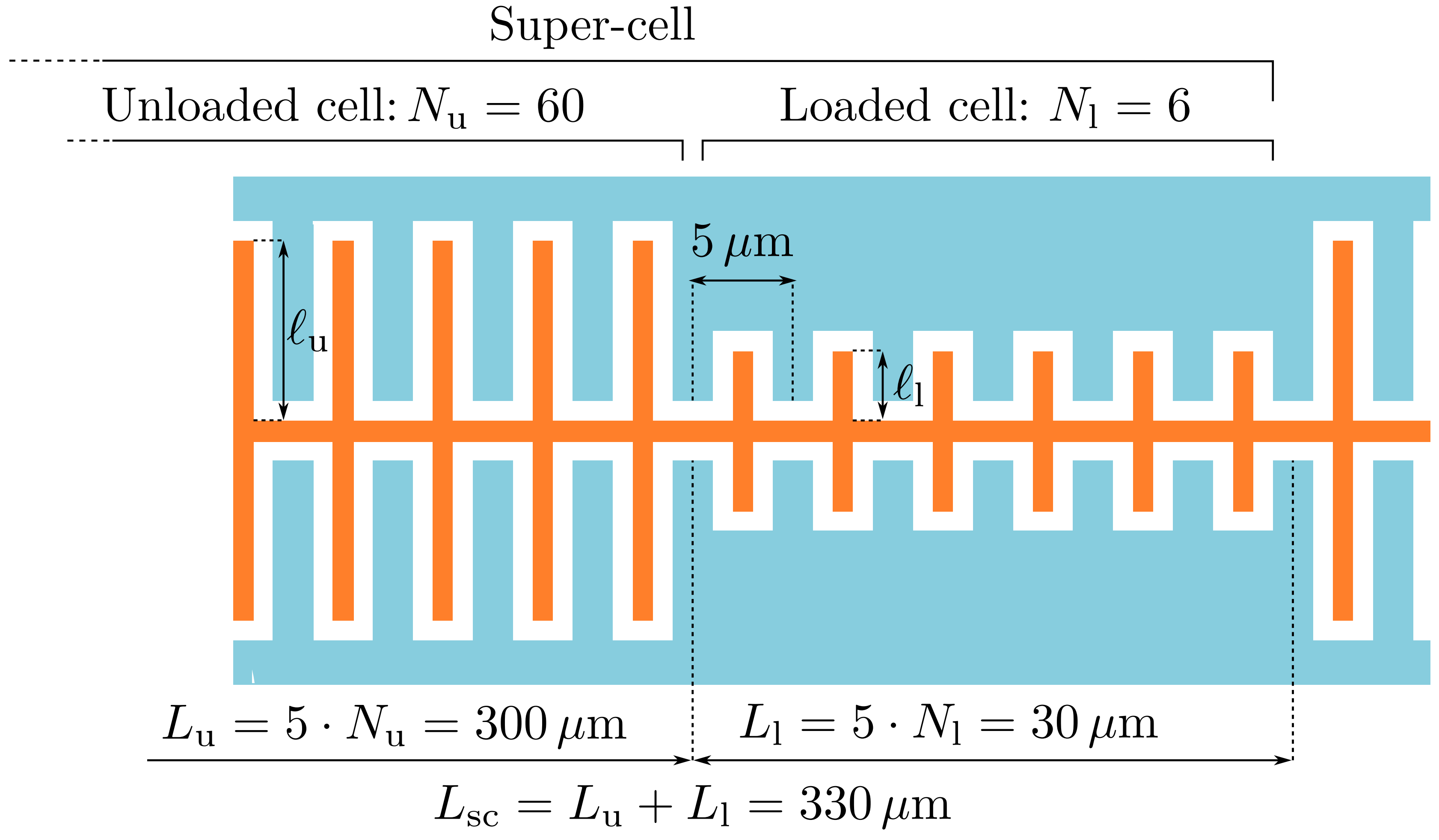
$$\frac{\partial I_u}{\partial x} = i\frac{\epsilon(\omega_u)}{4}\frac{k_u}{t_u}\left(\mathcal{F}^{s\mp c}_{u}t_s t_c I_s I_c + \mathcal{F}^{c_2 d}_{u}t_{c_2}t_d I_{c_2}I_d\right) + i\frac{\xi(\omega_u)}{8}k_u I_u\left(\mathcal{F}^{uuu^*}_{u}|t_u|^2|I_u|^2 + 2\mathcal{F}^{u\pm a\pm a^*}_{u}|t_a|^2|I_a|^2 + 2\mathcal{F}^{u\mp c\mp c^*}_{u}|t_c|^2|I_c|^2 + \sum_{m\in\{s,i,d,c_2\}}2\mathcal{F}^{umm^*}_{u}|t_m|^2|I_m|^2\right) + i\frac{\xi(\omega_u)}{8}\frac{k_u}{t_u}\left(\mathcal{F}^{d\mp c\mp c}_{u}t_d t_c^2 I_d I_c^2 + 2\mathcal{F}^{c_2\mp c^* s}_{u}t_{c_2}t_c^* t_s I_{c_2}I_c^* I_s + 2\mathcal{F}^{\pm ai^*\mp c}_{u}t_a t_i^* t_c I_a I_i^* I_c\right) \tag{S23}$$

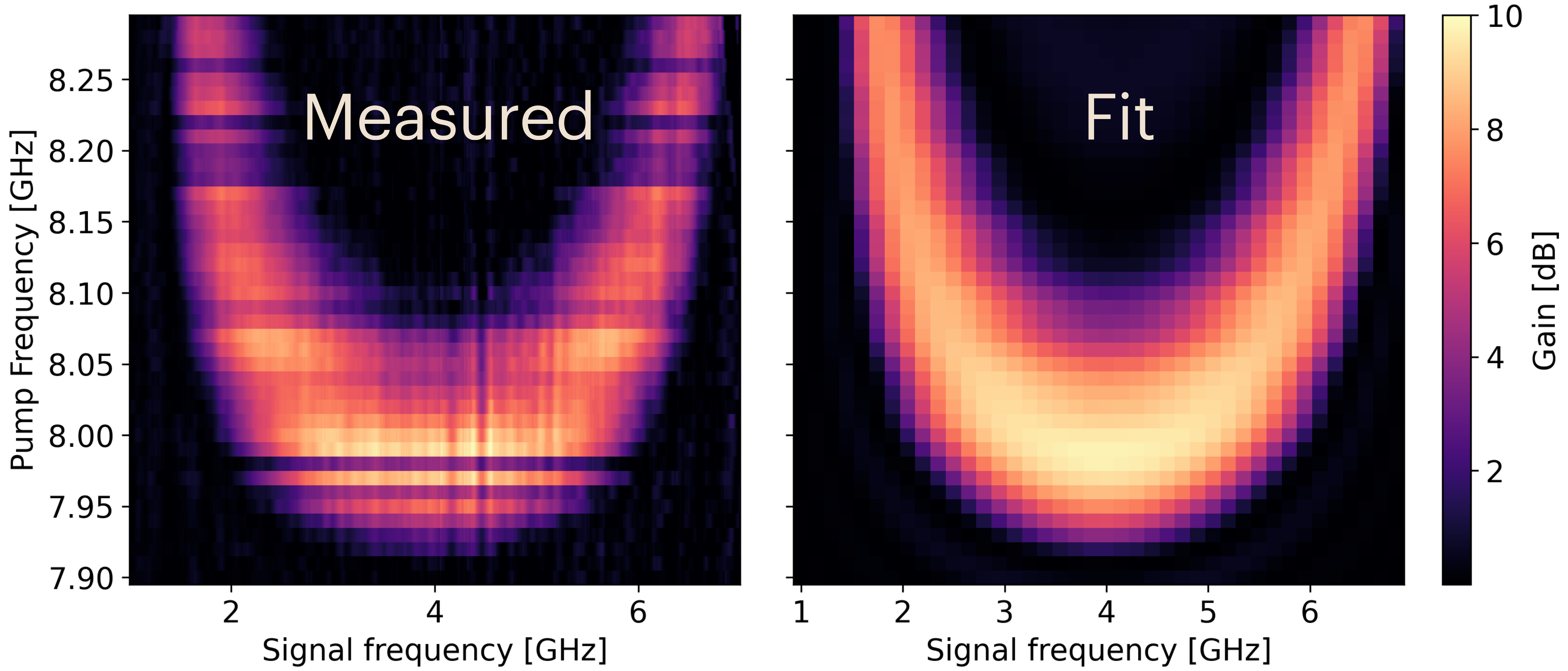
EXAMPLE: FIT OF DATA FROM FBK PROTOTYPE

- KI-TWPA prototypes based on a **CPW** have been fabricated at **Fondazione Bruno Kessler (FBK)** and also characterized at **Unimib.**
- Measured gain (≈ 9.2 dB), bandwidth (≈2 GHz) and added noise photons (2.5-5) compatible with fabrication target.







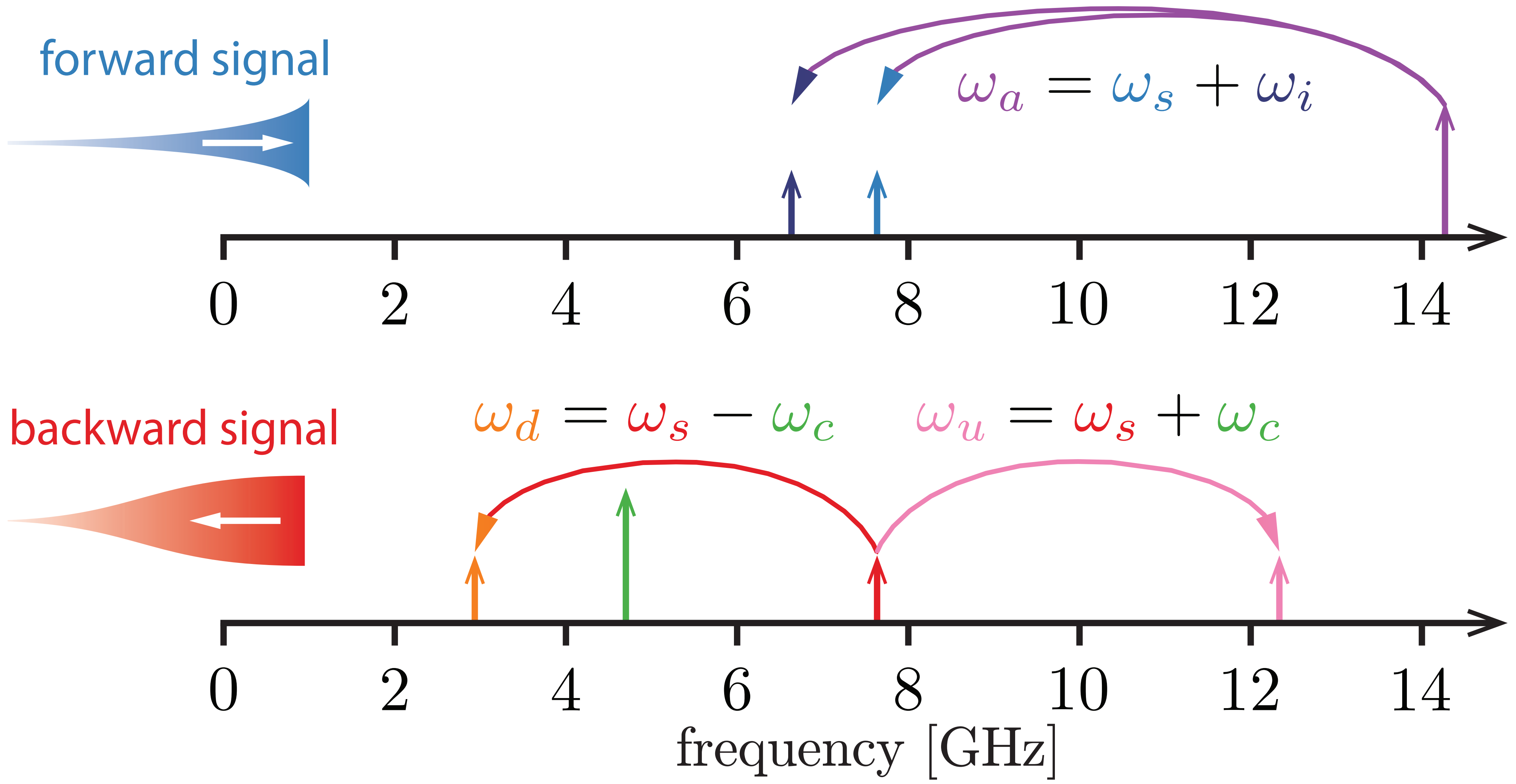


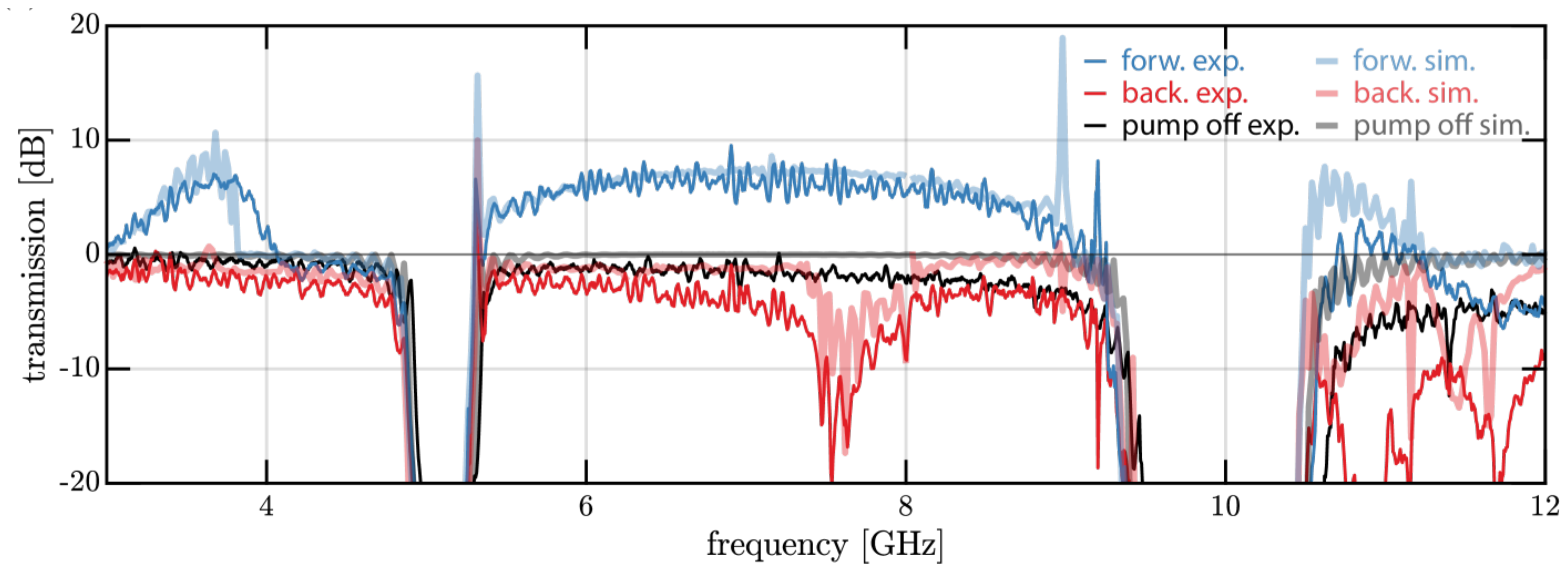
- The stopband position and optimal pump frequency are slightly different than expected.
- This can be attributed to **variations of the film thickness,** resulting in a **different** L_k.
- Simple 3WM gain model as a function of L_k fits the data reasonably well, predicts a ≈7% difference in L_k from expected.

EXAMPLE: TWPA AND CONVERTER

Main challenge for KI-TWPAS: isolate qubits/sensors from the pump

- Very **high pump power** (-30 to -40 dBm range) required.
- Amplification is bidirectional.
- Reflections from impedance mismatches and pump leakage can be disruptive.

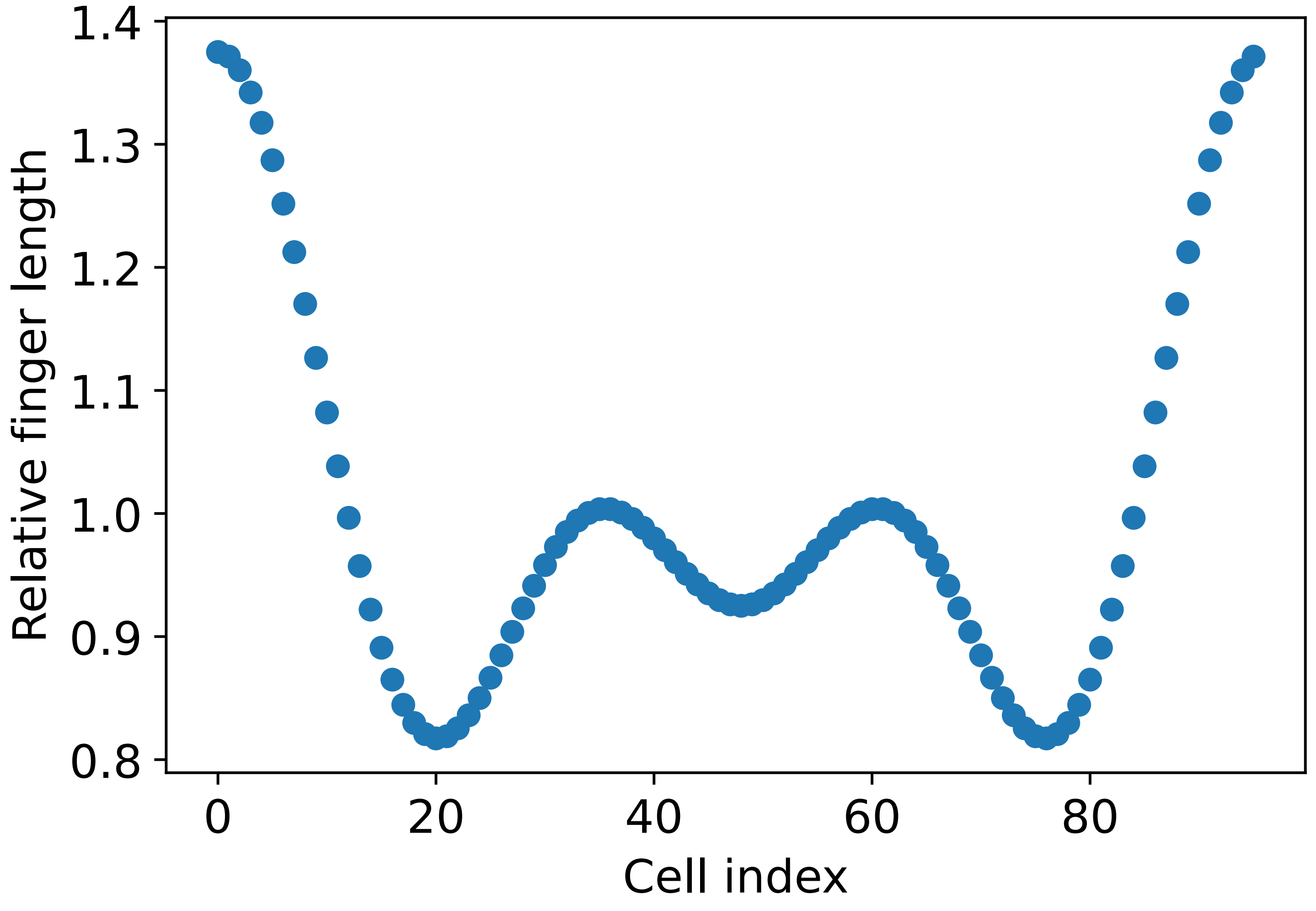




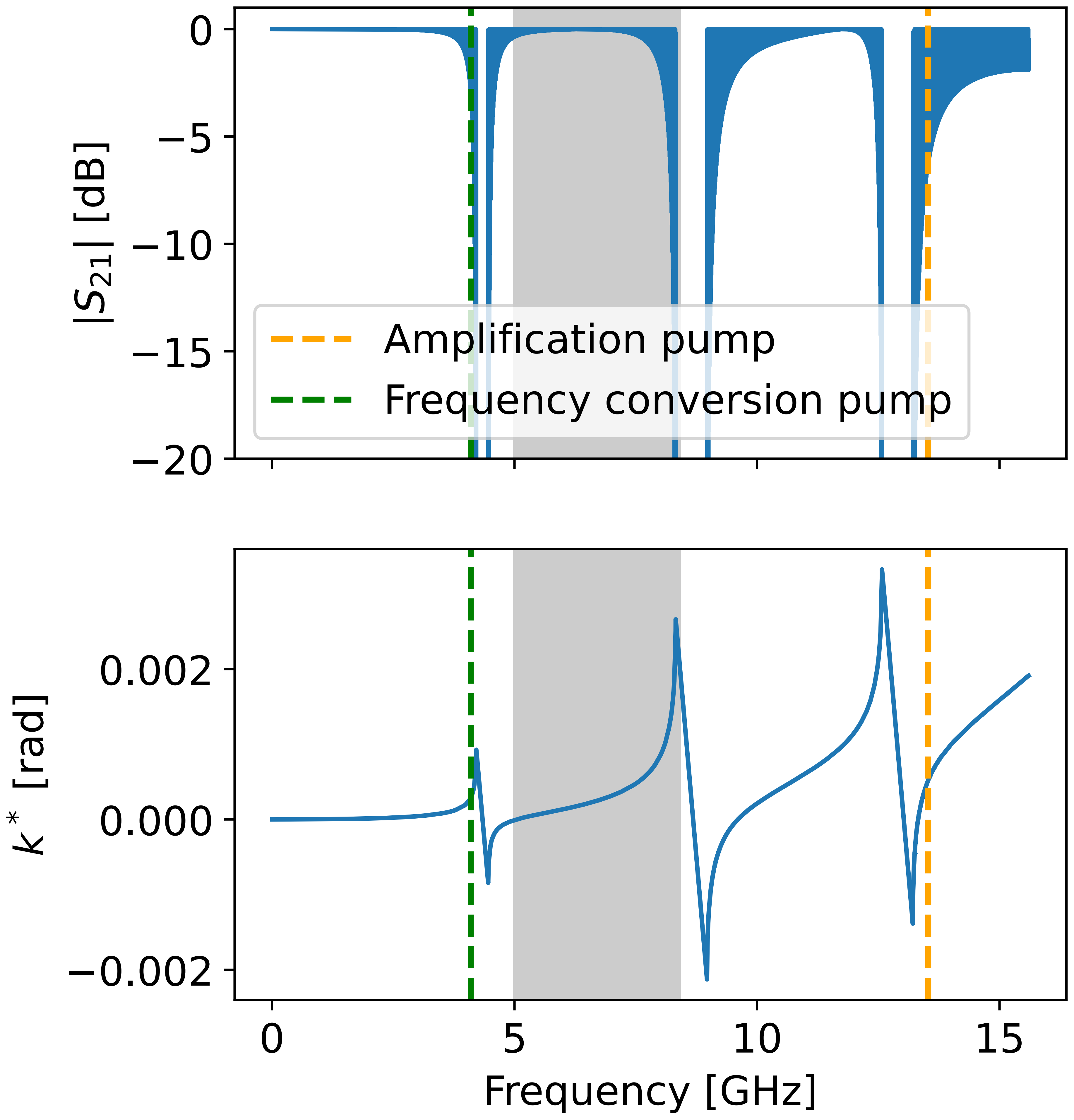
- Recently proposed design using JJs: Malnou, M., et al. "A traveling-wave parametric amplifier and converter." arXiv preprint (2024).
- Achieves **forward amplification** and **backward isolation**.
- The **forward-travelling pump** provides 3WM amplification.
- The **backward pump** engages a 3WM frequency conversion process that removes signal from the amplification band.

PRELIMINARY: A KI-TWPA AND CONVERTER

- First attempt to create a TWPA-C design **based on kinetic inductance**.
- Started from **realistic circuit parameters** used in KI-TWPA devices developed at NIST.

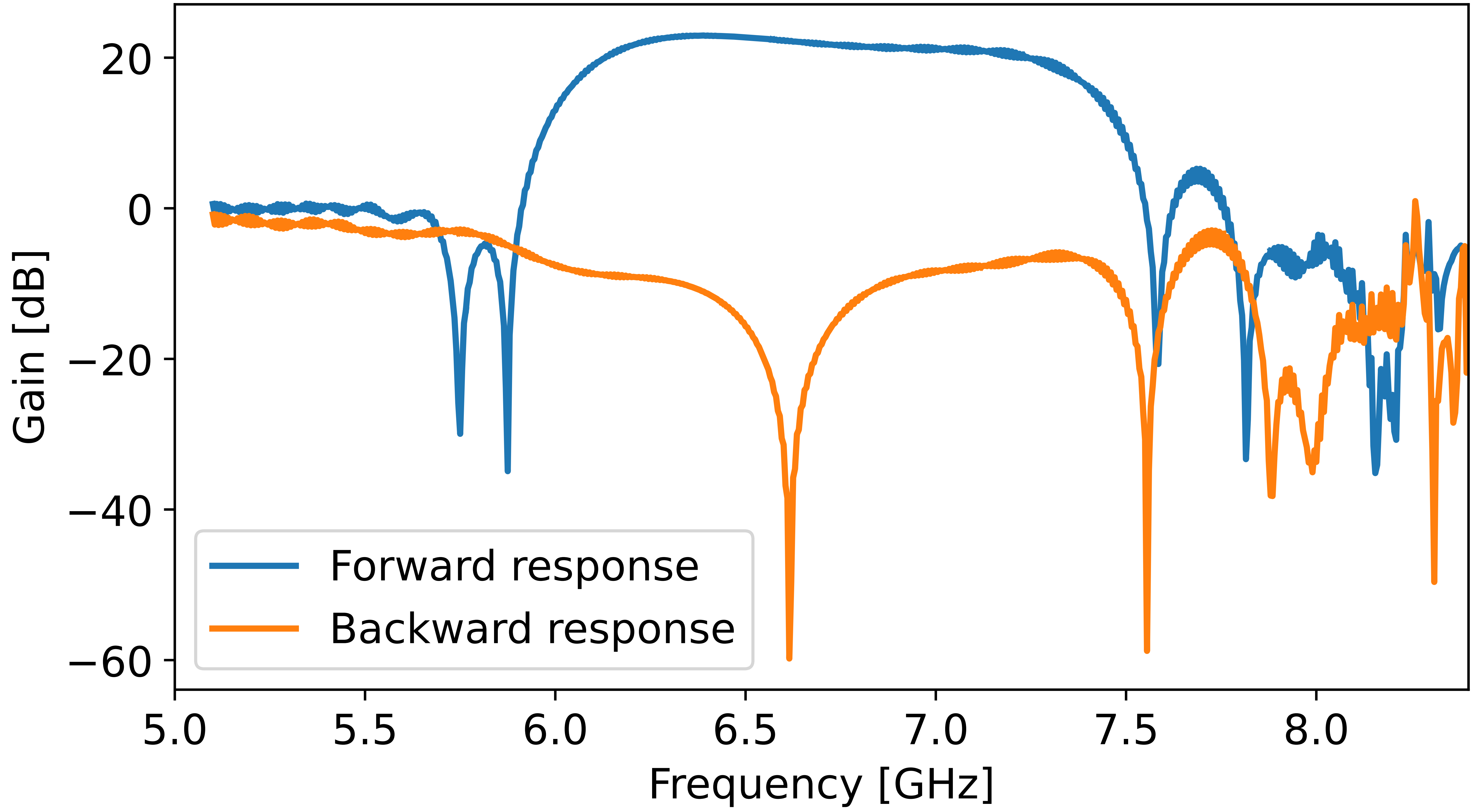


- The finger length is modulated according to three $\cos^2$ oscillations, creating stopbands with **different widths.**
- The frequency conversion pump is placed before the first stopband and the amplification pump after the third one.
- The wider second stopband limits the generation of the FC pump first harmonic.



PRELIMINARY: A KI-TWPA AND CONVERTER

- The forward and backward response are simulated using the generalized CMEs system and considering seven modes.
- Achieves ≈20 dB forward gain and average ≈10 dB backward isolation across the amplification bandwidth.
- Bandwidth is kinda limited, around 1.5 GHz



CONCLUSIONS

- Integrated design and simulation of KI-TWPAs in comprehensive framework.
- Faster EM simulations using even-odd analysis to extrapolate response of long periodic structure from single cell.
- Generalized Coupled Mode Equations solver for realistic gain response and simulation of complex devices.

TODO

- Systematic study of simulated vs measured responses.
- Improve design of novel KI-TWPA-Converter device.
- Implement optimizer to reach target designs even faster.

COLLABORATION

UNIMIB/INFN-MIB group

Pietro Campana, Matteo Borghesi, Rodolfo Carobene, Alessandro Cattaneo, Hervè Corti, Marco Faverzani, Elena Ferri, Sara Gamba, Marco Gobbo, Logan Howe, Danilo Labranca, Roberto Moretti, Angelo Nucciotti, Luca Origo, Andrea Giachero

FBK and University of Trento group

Felix Ahrens, Nicolò Crescini, Paolo Falferi, Benno Margesin, Federica Mantegazzini, Renato Mezzena, Andrea Vinante, Enrico Bogoni (also Unimib), Marcello Faggionato (also Unimib), Alessandro Irace (also Unimib)

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY (NIST)

Logan Howe