

Spoke 6 Quantum Integration

Quantum Collective States in Superconducting 5-Qubit Networks



B. Ruggiero MC E. Esposito MC C. Bonavolontà TD MC

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M. Cirillo



PNRR Partenariato Esteso
NATIONAL QUANTUM SCIENCE AND TECHNOLOGY INSTITUTE "NQSTI"

Spoke 6 Quantum Integration - The Role of ISASI

Quantum Collective States in Superconducting Qubit Networks (SQN)

	Scientific Item	Milestones		MC TI CNR	
6.3.3 Superconducting quantum networks	6.3.3.2 Demonstration of superconducting quantum networks topologies composed of interacting Josephson devices in 2D lattices.	M12-A6.3 M24-A6.3 M36-A6.3	CNR INFN	B. Ruggiero (CNR ISASI, NA) M. Lisitskiy (CNR-SPIN, NA), M. Salluzzo (CNR SPIN, NA), E. Esposito (CNR ISASI, NA), C. Gatti (INFN LNF)	

2023 - 2025

**Quantum Integration 400 KEuro
(Spoke 6 ISASI -Pozzuoli)**

SQN Superconducting Qubit Network

2022 2024

PHYSICAL REVIEW B **105**, 104516 (2022)



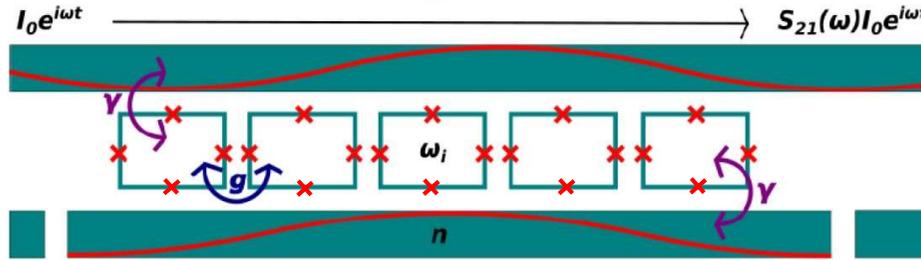
Quantum dynamics of disordered arrays of interacting superconducting qubits quantum collective states

M. V. Fistul^{1,2}, O. Neyenhuys¹, A. B. Bocaz¹, M. Lisitskiy³ and I. M. Eremin¹

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$$\hat{H}_{\text{tot}} = \hat{H}_{\text{SQA}} - \alpha I(t) \sum_i \hat{\sigma}_i^z,$$

$$\hat{H}_{\text{SQA}} = \hat{H}_{\text{qb}} + \hat{H}_{\text{SR}} + \hat{H}_{\text{LR}} + \hat{H}_{\text{ph}} + \hat{H}_{\text{qb-ph}},$$

$$\hat{H}_{\text{qb}} = \sum_{i=1}^N \left[\frac{\Delta_i}{2} \hat{\sigma}_i^x + \frac{\epsilon_i}{2} \hat{\sigma}_i^z \right].$$

α are the coupling strength between qubits and the transmission line
 $I(t)$ the current flowing along the transmission line

\hat{a}^\dagger (\hat{a}) are the creation (annihilation) photon operators

$$\hat{H}_{\text{SR}} = g_{\text{SR}} \sum_j \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z,$$

$$\hat{H}_{\text{ph}} = \hbar \omega_0 \hat{a}^\dagger \hat{a},$$

$$\hat{H}_{\text{LR}} = g_{\text{LR}} \sum_{j=1,2,3,4} [\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y],$$

$$\hat{H}_{\text{qb-ph}} = \gamma \sum_{i=1}^N \hat{\sigma}_i^z (\hat{a}^\dagger + \hat{a}),$$

transmission coefficient

$$\Delta S_{21}(\omega) \simeq C(\omega),$$

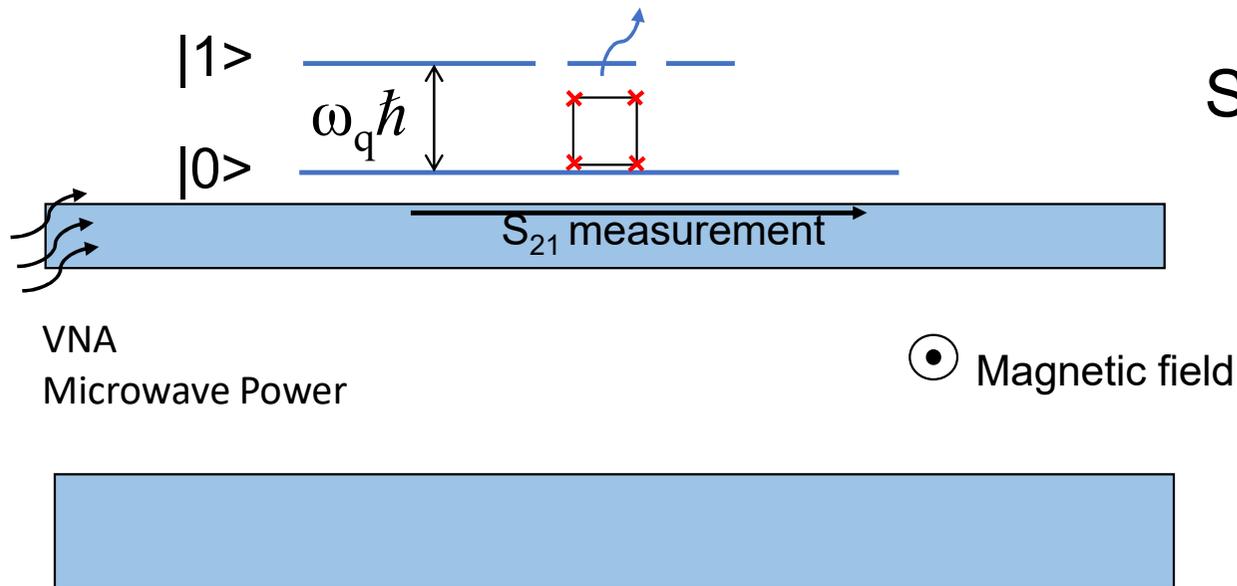
dynamic susceptibility,

The schematic of an SQN with five flux qubits coupled to the low-dissipative cavity and the transmission line is presented. The transmission line is used to establish an experimental setup for the measurements of S21

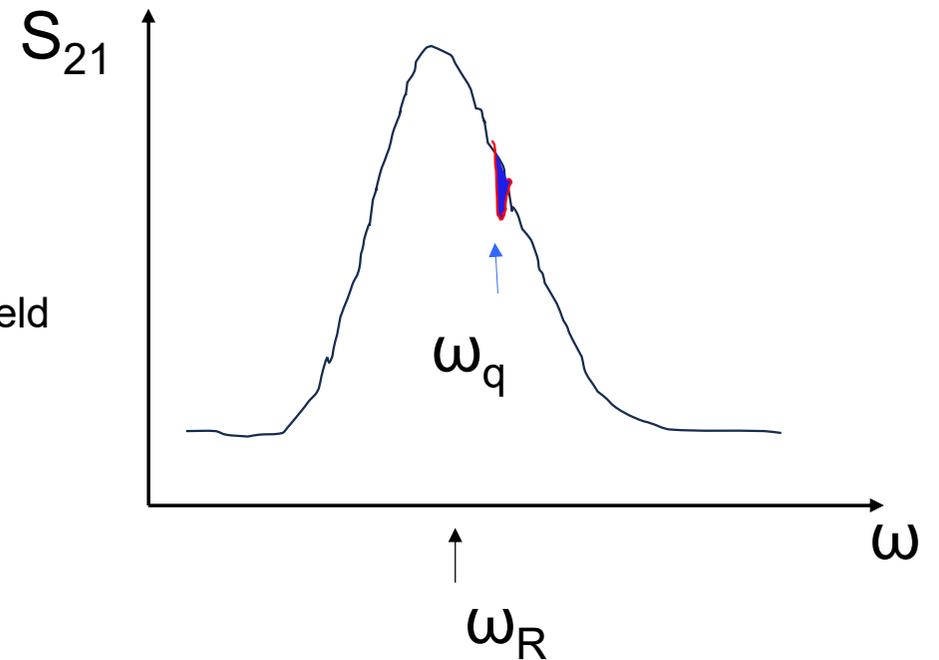
The transmission coefficient S_{21} vs frequency behaviour could present dips (absorption) when photons are exchanged in the resonant cavity

Nonlinear interactions between photons of energy level transitions in qubits and of microwave power

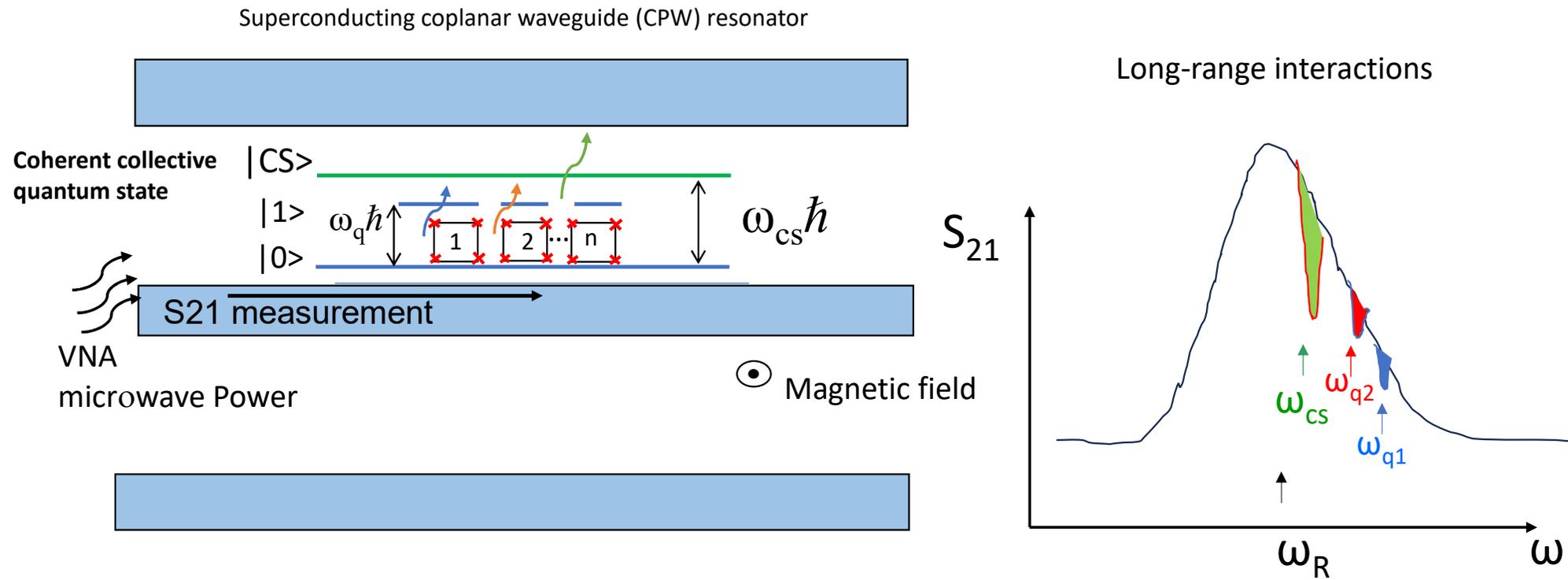
Superconducting coplanar waveguide (CPW) resonator



S_{21} vs ω
transmission coefficient as function of frequency



Superconducting qubit network (SQN) with 5 flux qubits with 4 JJ



Oscillations on S_{21} indicate the interaction between qubit and resonant circuit.

Main resonant dip..... is associated with COLLECTIVE QUANTUM STATE in SQN.

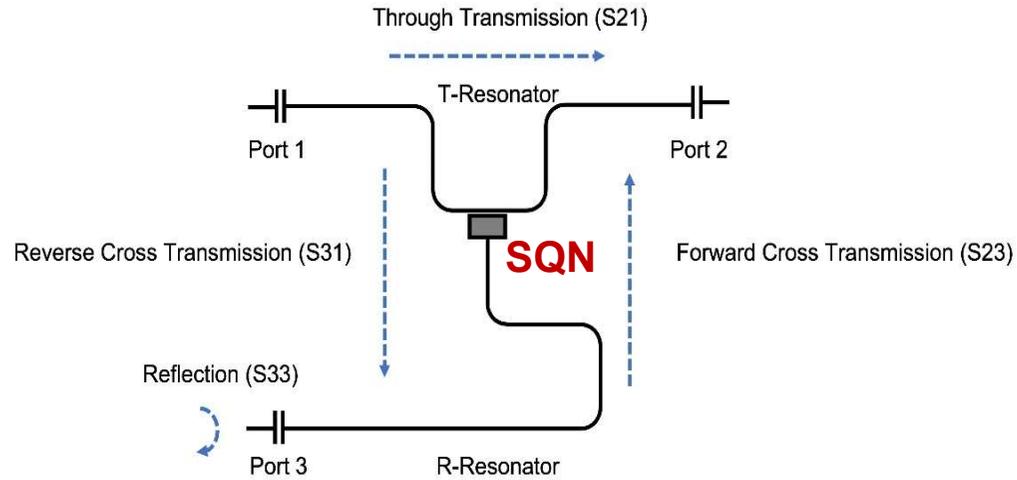
Experiments on two Resonators with SQN with 5 flux qubits

G. Brida and E. Enrico

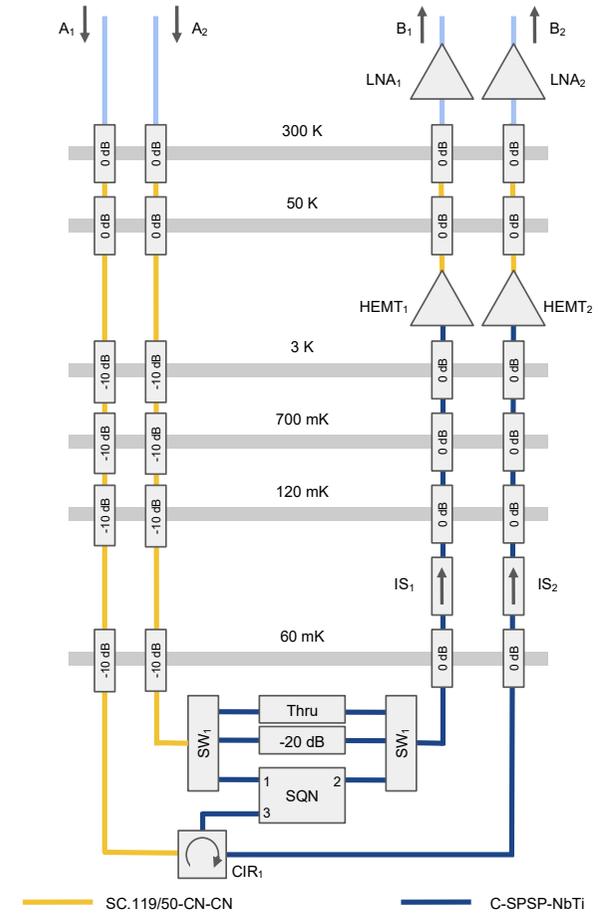
2023/24



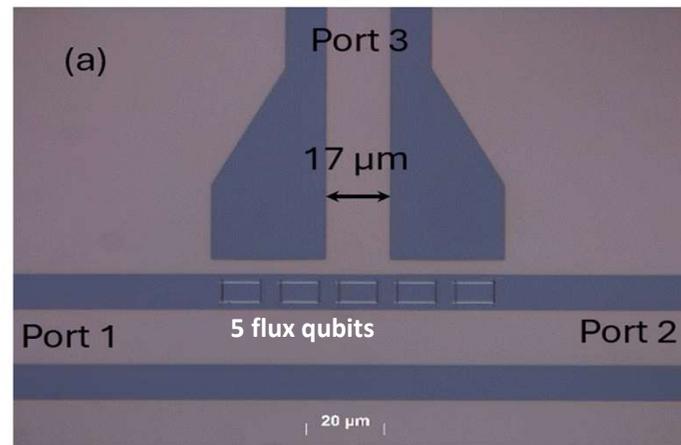
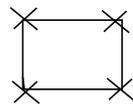
CNR SPIN Pozzuoli
CNR ISASI Pozzuoli



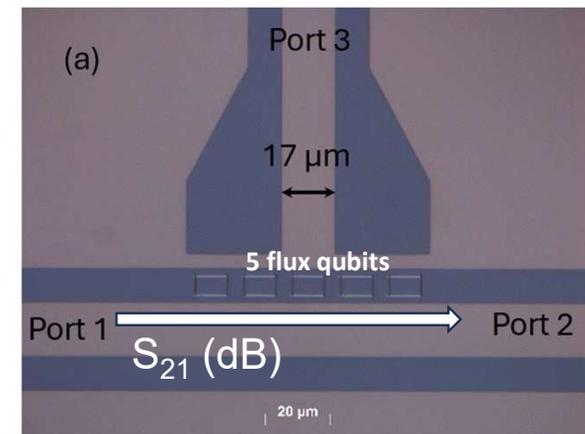
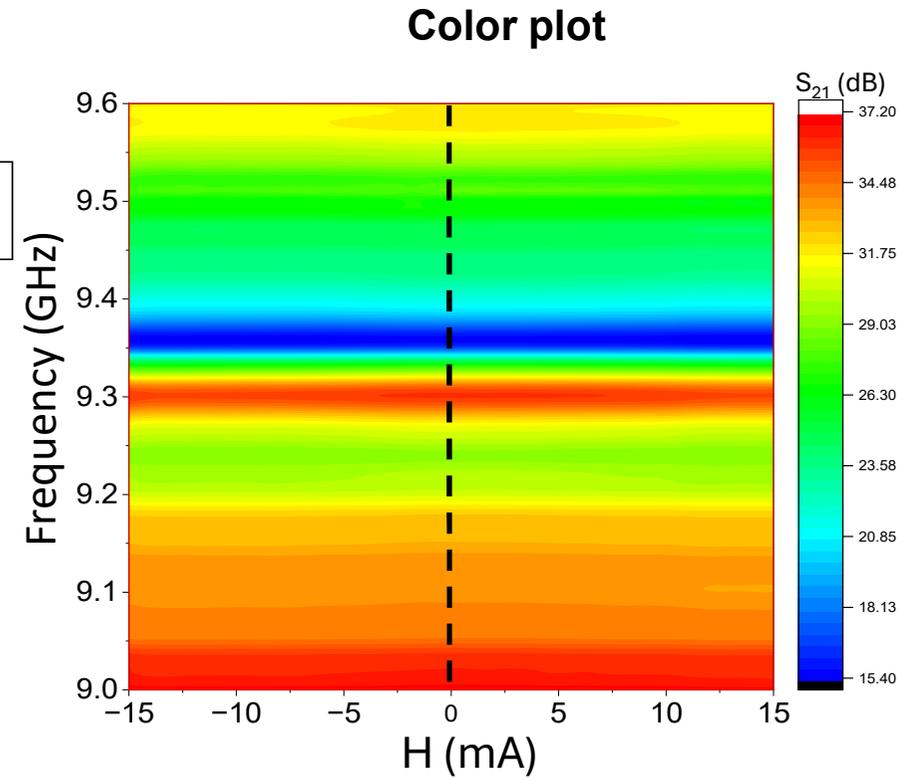
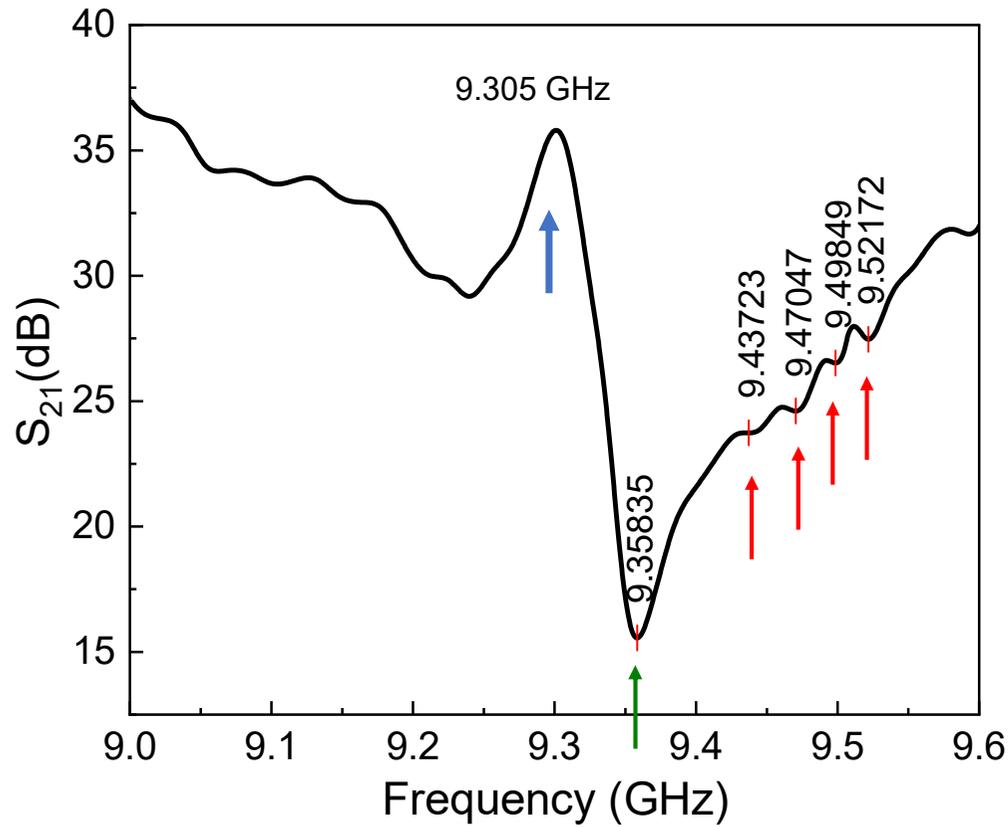
T = 20 mK



4 JJ flux qubits

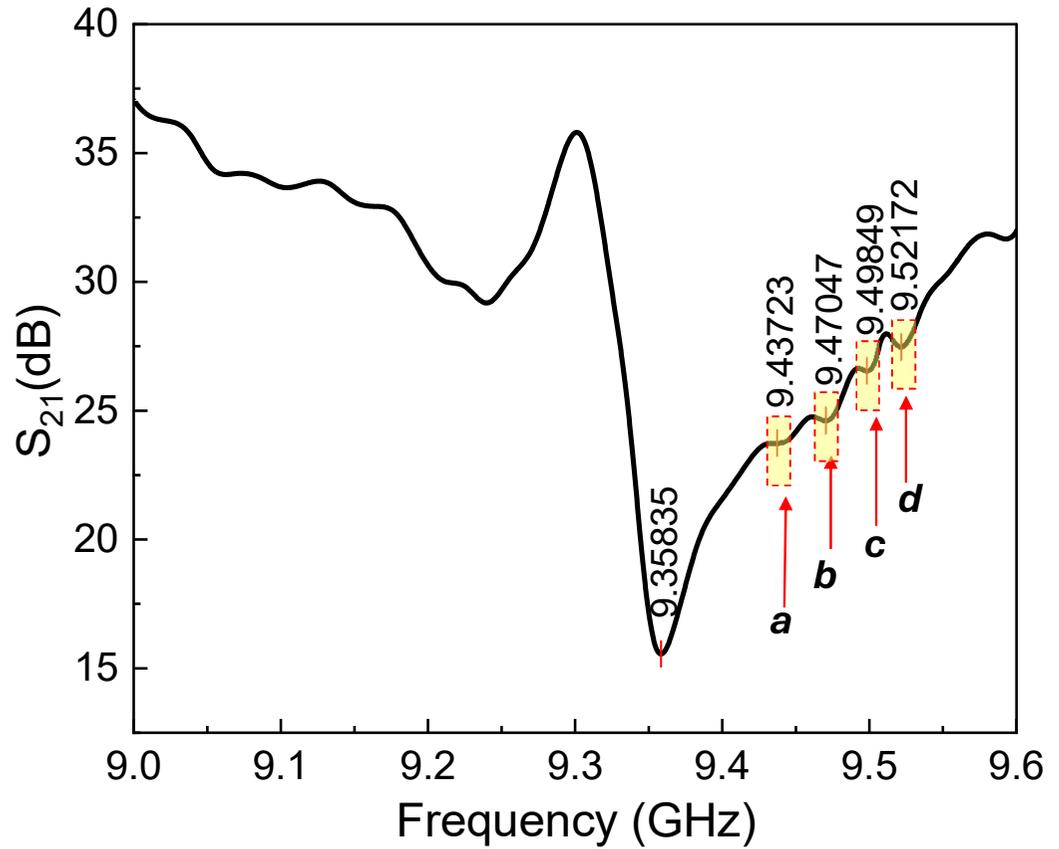
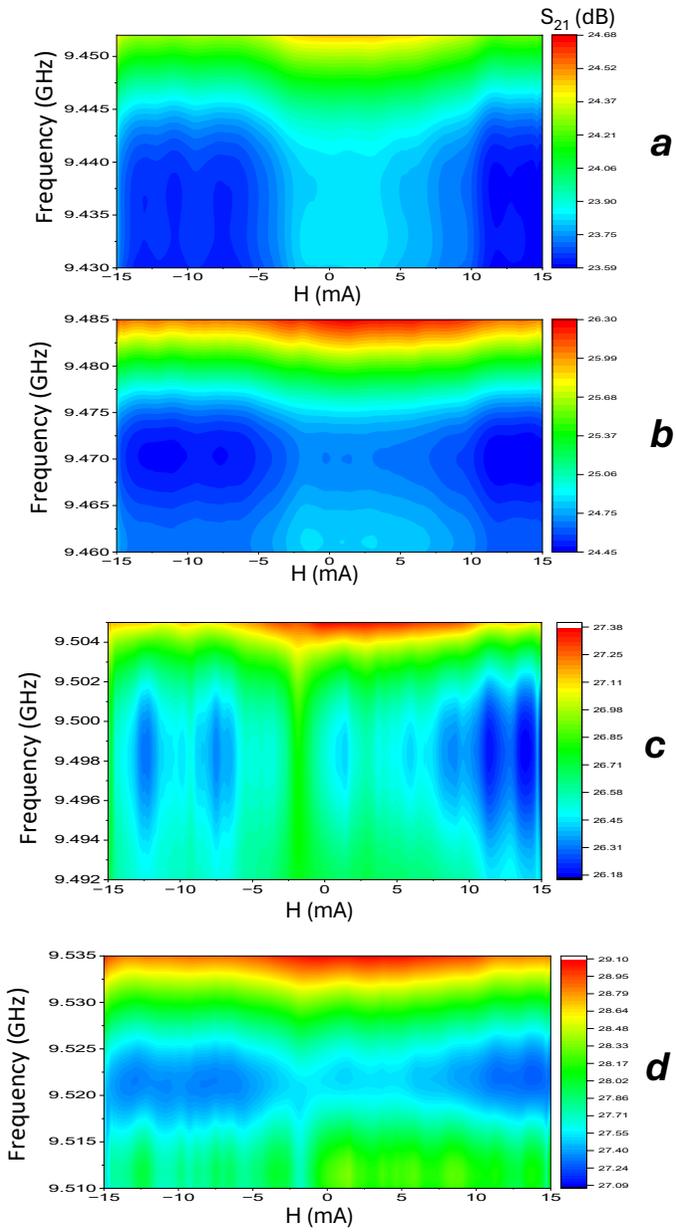


The transmission measurements of S_{21} versus frequency demonstrate the presence of **number of resonant dips**.

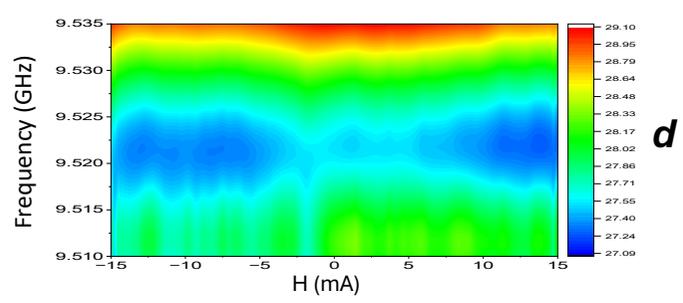
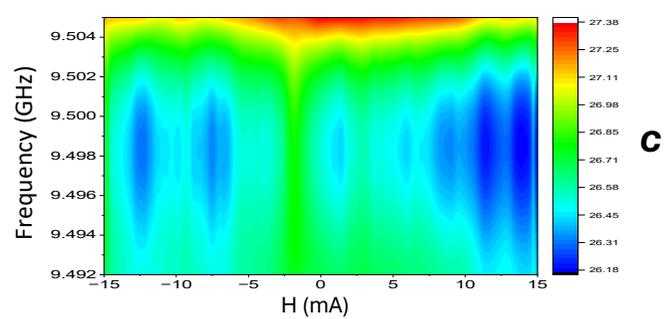
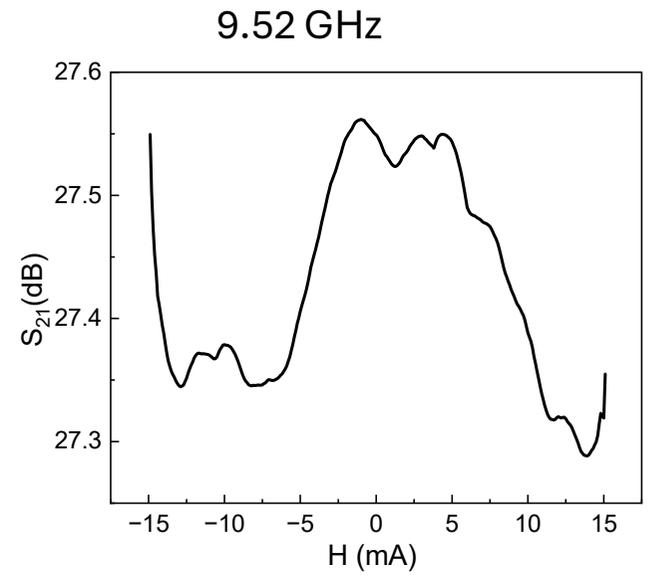
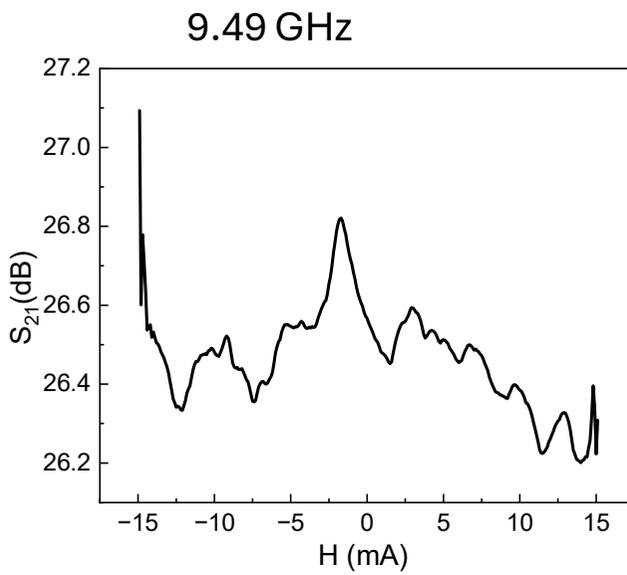
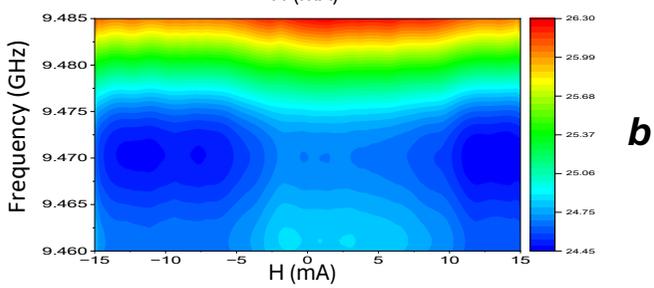
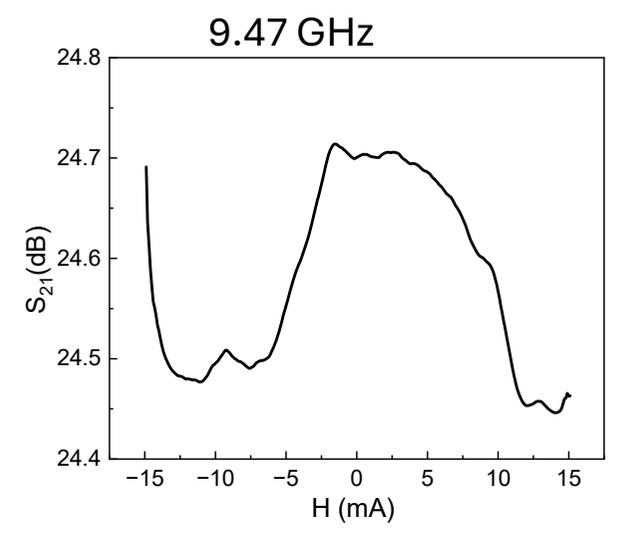
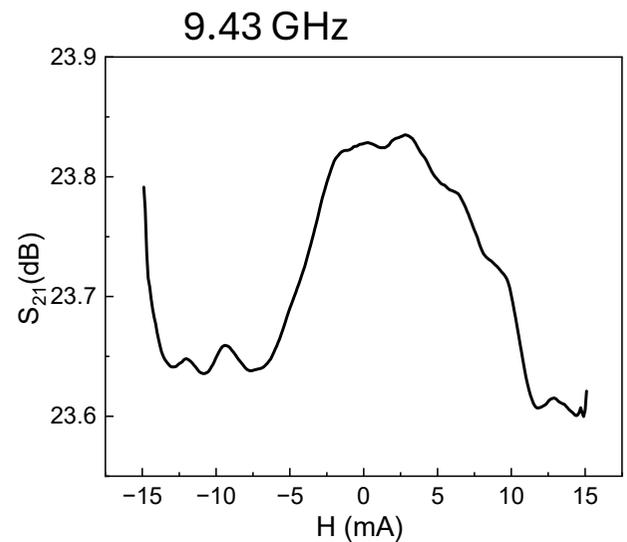
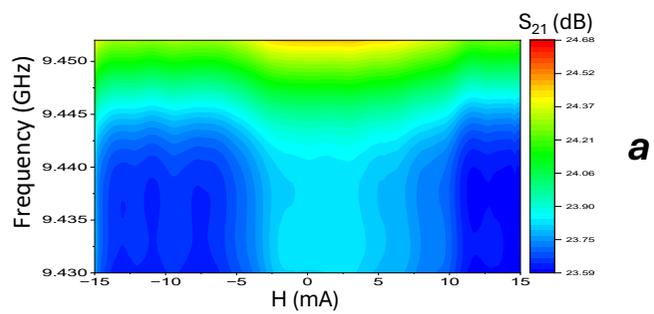


Periodic dependence of resonant dips as function of the external magnetic field is associated with **the excitation of single qubits**

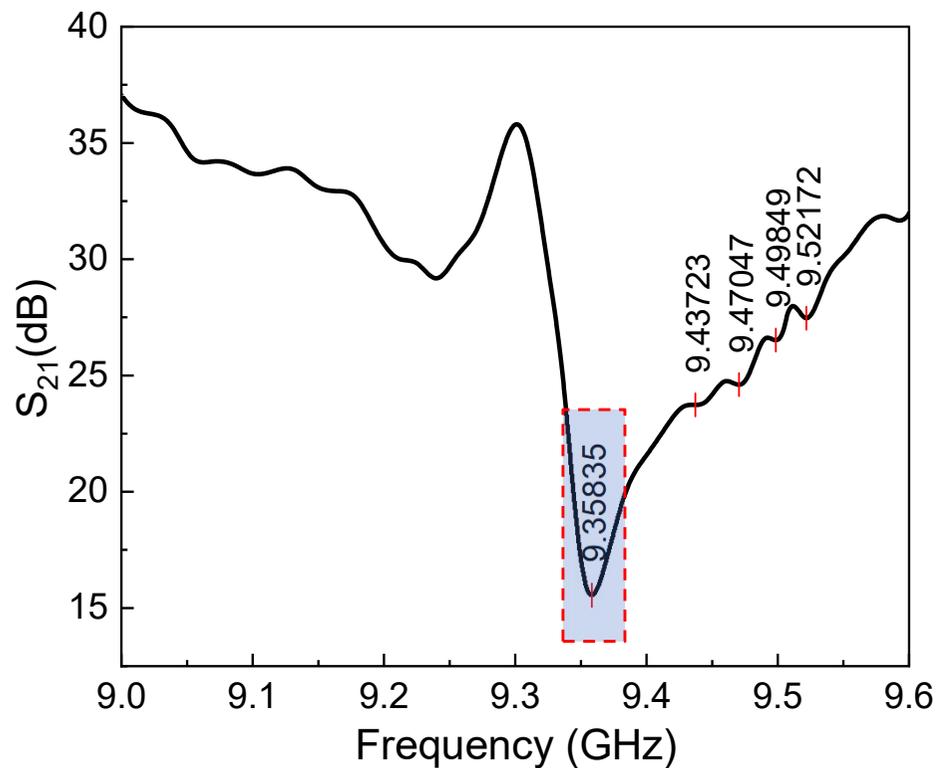
S_{21} resonance peaks as function of external magnetic field



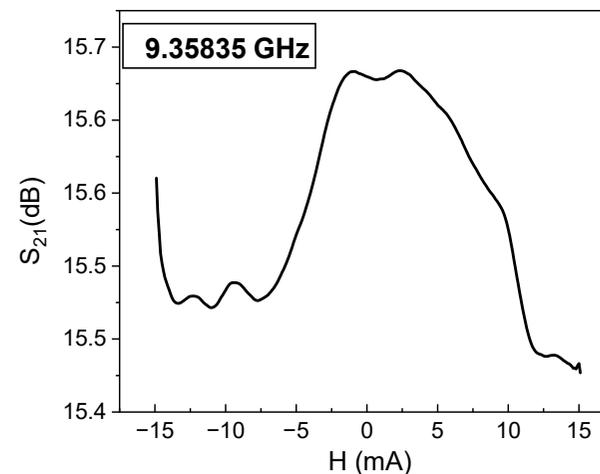
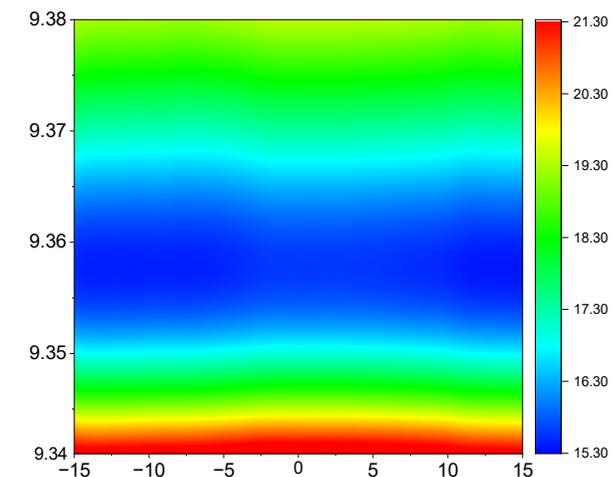
Periodic dependence of resonant dips as function of the external magnetic field is associated with **the excitation of single qubits**



3 terminal SQN with 5 flux qubits



Color map S_{21} main dip resonance 9,35835 GHz



<https://nqsti.it/research-highlights> March 27th, 2024

The main resonant dip in $S_{21}(\omega)$ dependence and its periodic magnetic field oscillation indicate the presence of a strong long-range interaction between qubits leading to the quantum collective state

Is it possible to improve
the robustness of
Quantum Collective
States in SQN?



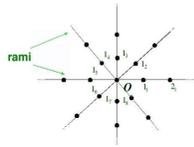
2 December 2024

The poster has a light blue background with a network of white lines and dots. At the top left is the NQSTI logo. To its right, a white box contains the text: 'PNRR Partenariato Esteso', 'NATIONAL QUANTUM SCIENCE AND TECHNOLOGY INSTITUTE "NQSTI"', and 'Spoke 6 Quantum Integration'. In the center, the title 'Workshop on Quantum Integration and Topology' is written in a bold, italicized serif font, with 'INO CNR Firenze, November 25th, 2024' below it. Below the title, another white box lists the 'Scientific Committee' members: 'Luca Pezzè - INO CNR', 'Berardo Ruggiero - ISASI CNR', and 'Augusto Smerzi - INO CNR'. At the bottom, there are three logos: on the left, the logo for 'Istituto Nazionale di Scienze e Tecnologie' (ISTeT); in the center, the 'CNR-INO' logo with 'ISTITUTO NAZIONALE DI OTTICA' and 'CONSIGLIO NAZIONALE DELLE RICERCHE' below it; and on the right, the logo for 'Consiglio Nazionale delle Ricerche' (CNR) and 'SPIN'.

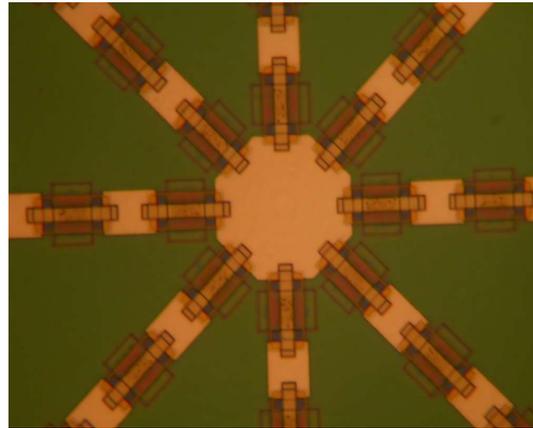
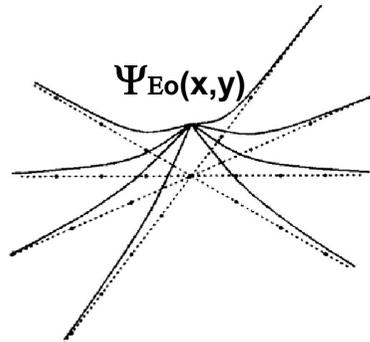
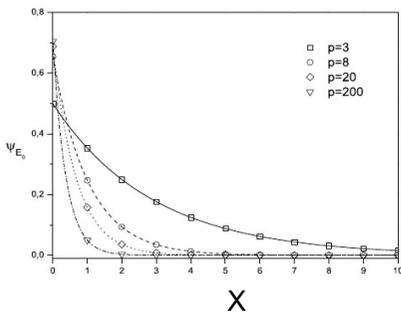
Workshop Quantum Integration and Topology

Topology-Induced Spatial Bose-Einstein Condensation in Inhomogeneous Josephson Junction Networks

JJ STAR NETWORK

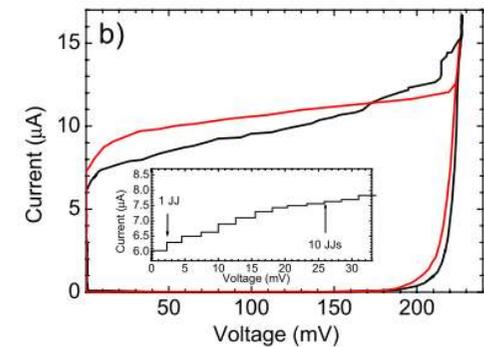
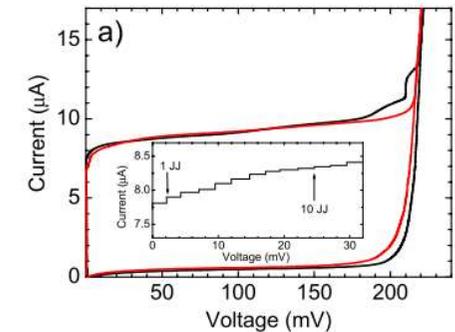


$$\psi_{E_0}^{star}(x, y) = \sqrt{\frac{p-2}{2p-2}} e^{-\frac{x}{\xi}} \quad \xi = \frac{2}{\log(p-1)}$$



The observed phenomena evidence new and surprising behavior of transport properties in superconducting Josephson junctions Network

Experiments: ISASI/Tor Vergata



$T = 1.2 \text{ k}$ $\Delta I_c = 15 \%$

P. Sodano, A. Trombettoni, P. Silvestrini, R. Russo, B. Ruggiero
Inhomogeneous superconductivity in comb-shaped Josephson junction networks
New Journal of Physics **8**, 327 (2006).

P. Silvestrini, R. Russo, V. Corato, B. Ruggiero, A. Trombettoni
Topology-induced critical current enhancement in Josephson networks.
Physics Letters A **370**, 499 (2007).

M. Lucci, V. Campanari, D. Cassi, V. Merlo, F. Romeo, G. Salina, and M. Cirillo
Quantum Coherence in Loopless Superconductive Networks.
Entropy, **24**, p.1690 (2022)

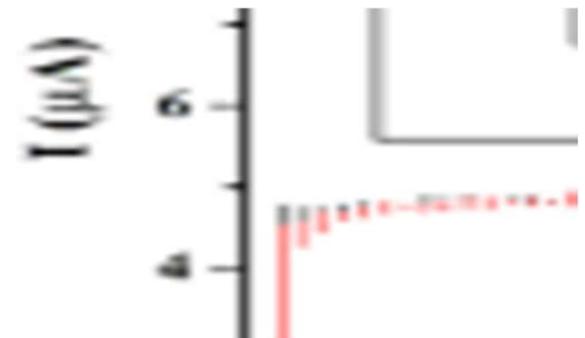
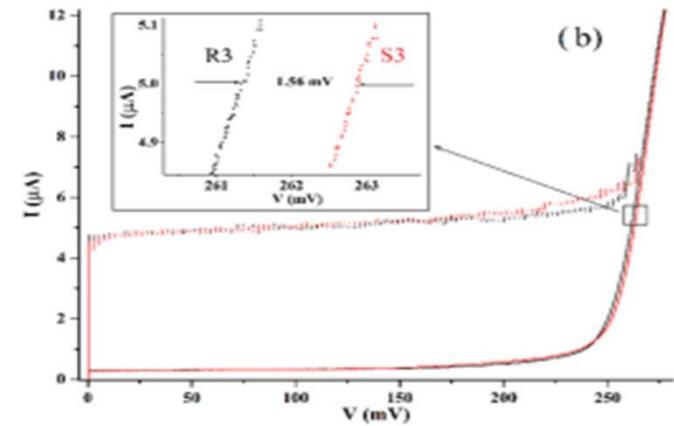
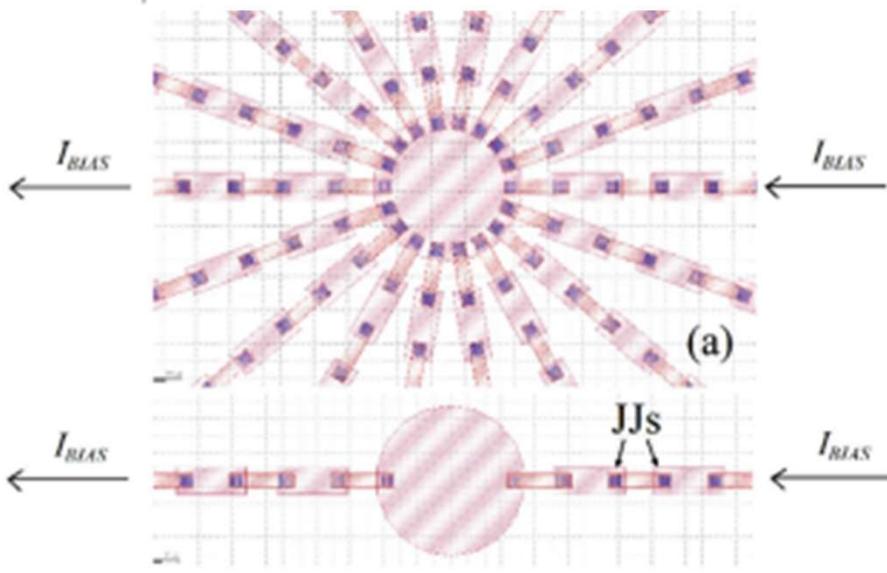
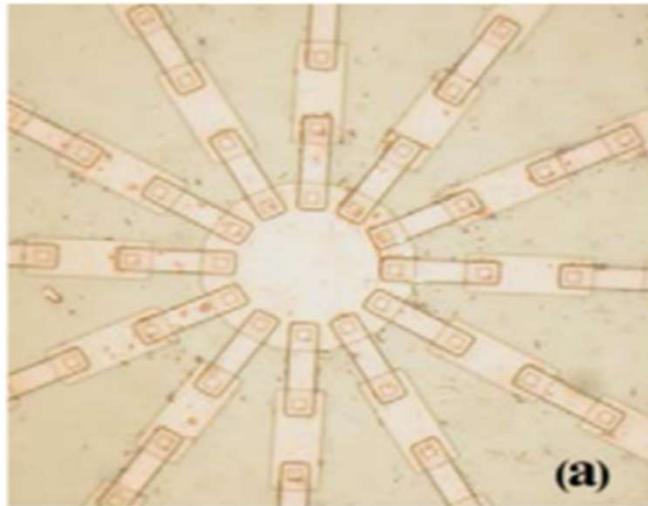
R. Bizzi, V. Campanari, D. Cassi, V. Merlo, and F. Romeo
Evidence of Long-Range Coherence in Superconducting Networks
IEEE Trans. On Appl. Supercond, **33**, p. 1800106, (2023)

Macroscopically Coherent patterns of wave functions along specific directions

Existence of a Bose Einstein Condensation

Enhancement of superconducting properties

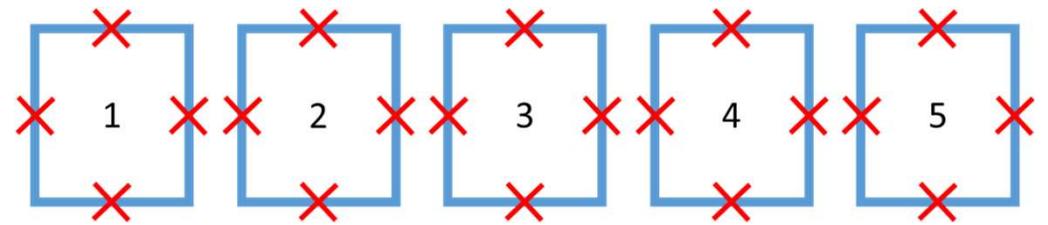
2025



$T = 4.2 \text{ K}$

$\Delta I_c = 8 \%$

Modeling Superconducting Qubit Networks with 5 flux qubit with inductive coupling



Assumptions

- “ferromagnetic” inductive coupling

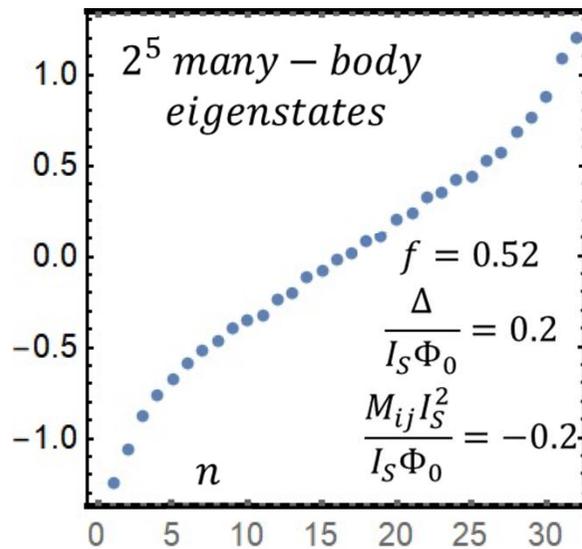
$$M_{ij} < 0$$

- nearest neighbors inductive coupling

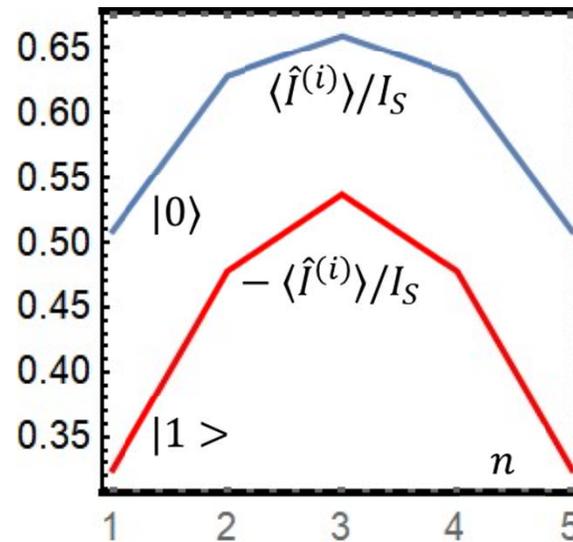
$$M_{ij} = 0, \text{ if } |i - j| > 1$$

- Absence of disorder

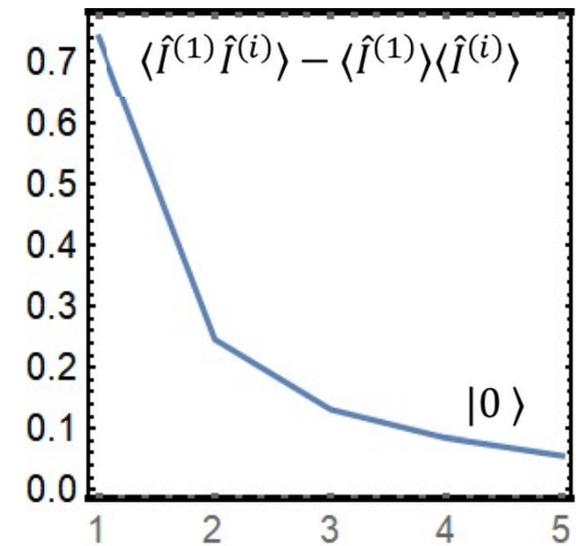
Energy eigenvalues (units $I_S \Phi_0$)



loop currents

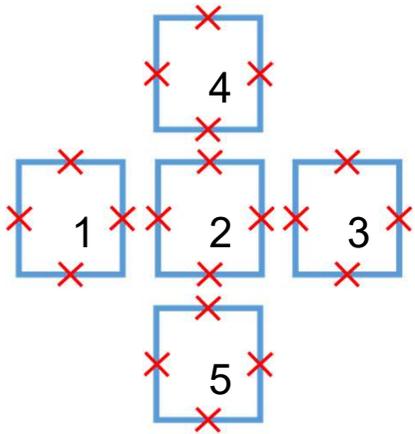


loop currents correlations

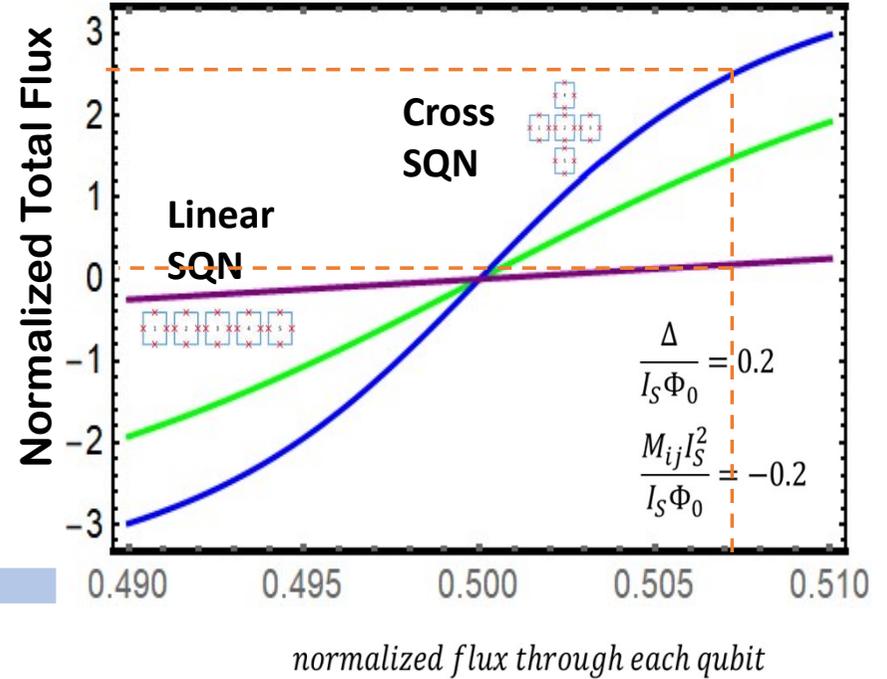
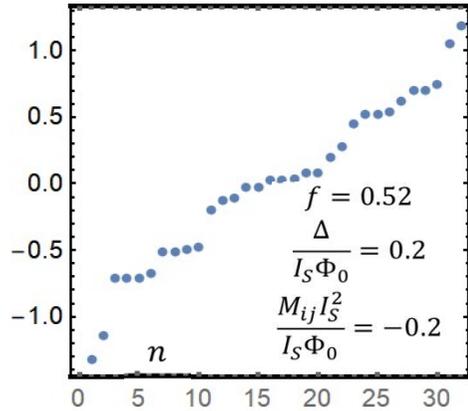


Insert Topology in Superconducting Qubit Networks with 5 flux qubit

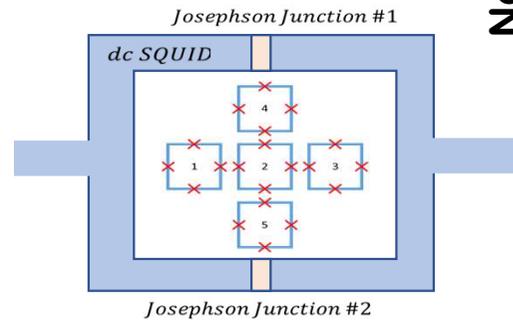
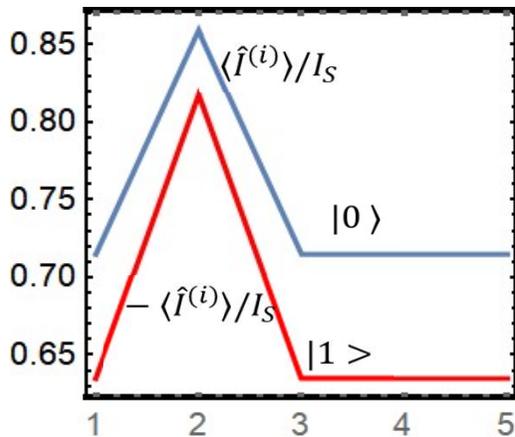
Cross SQN



Energy eigenvalues (units $I_S \Phi_0$)



loop currents



Topology-induced anharmonic spectral properties
Topology-induced loop currents enhancement

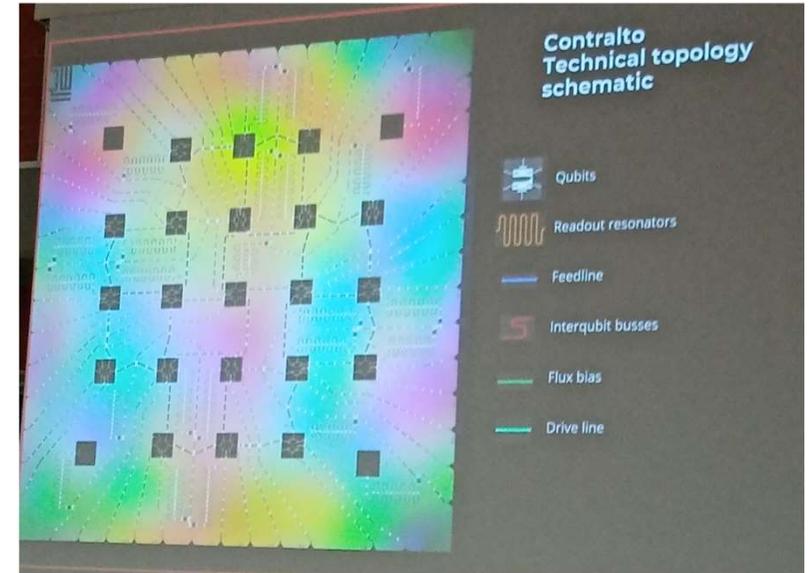
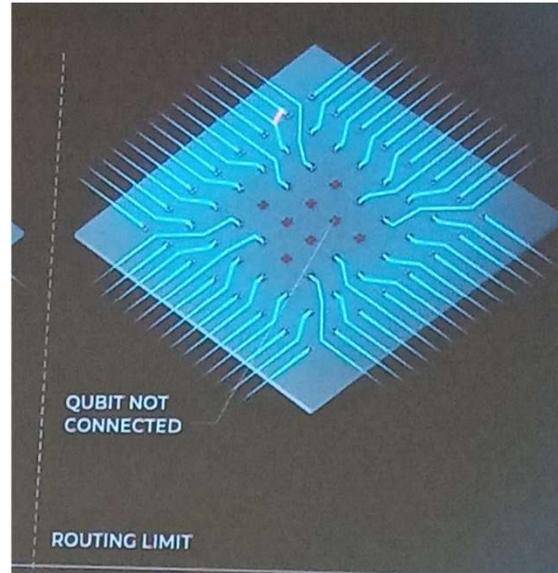
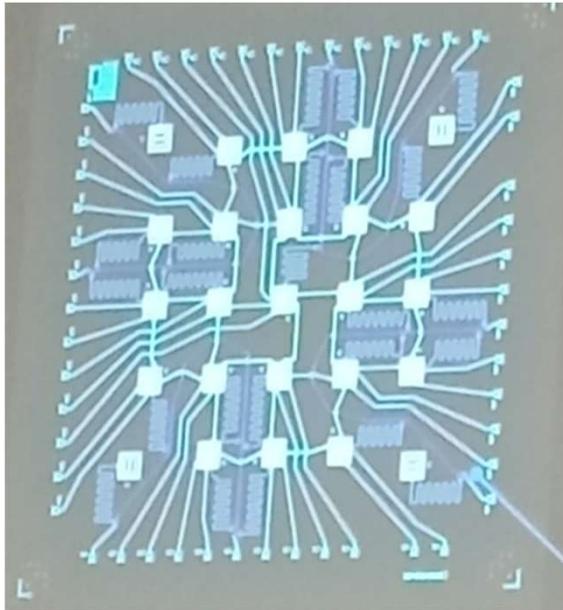


Robust QCS in SQN ?

Topological Corrections in realising qubit chip

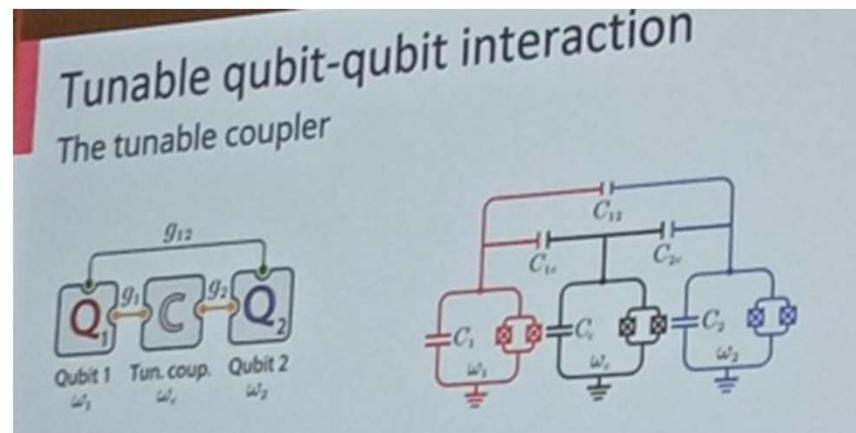
QC Companies

QuantumWare



Rigetti

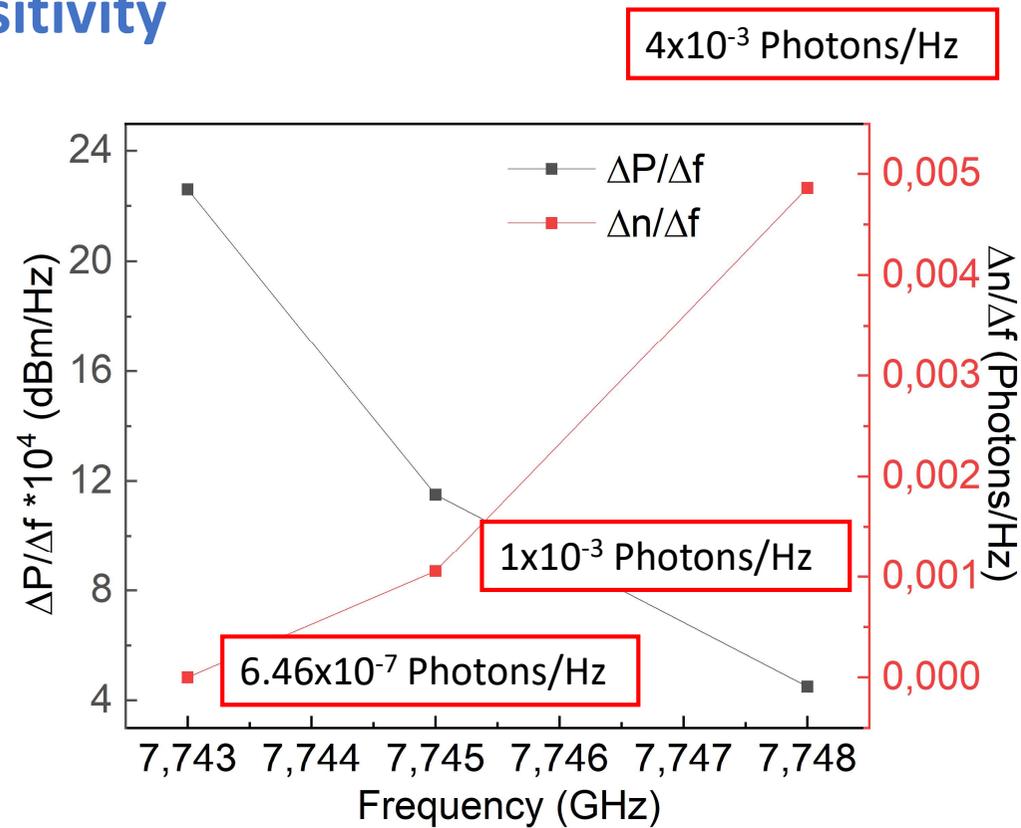
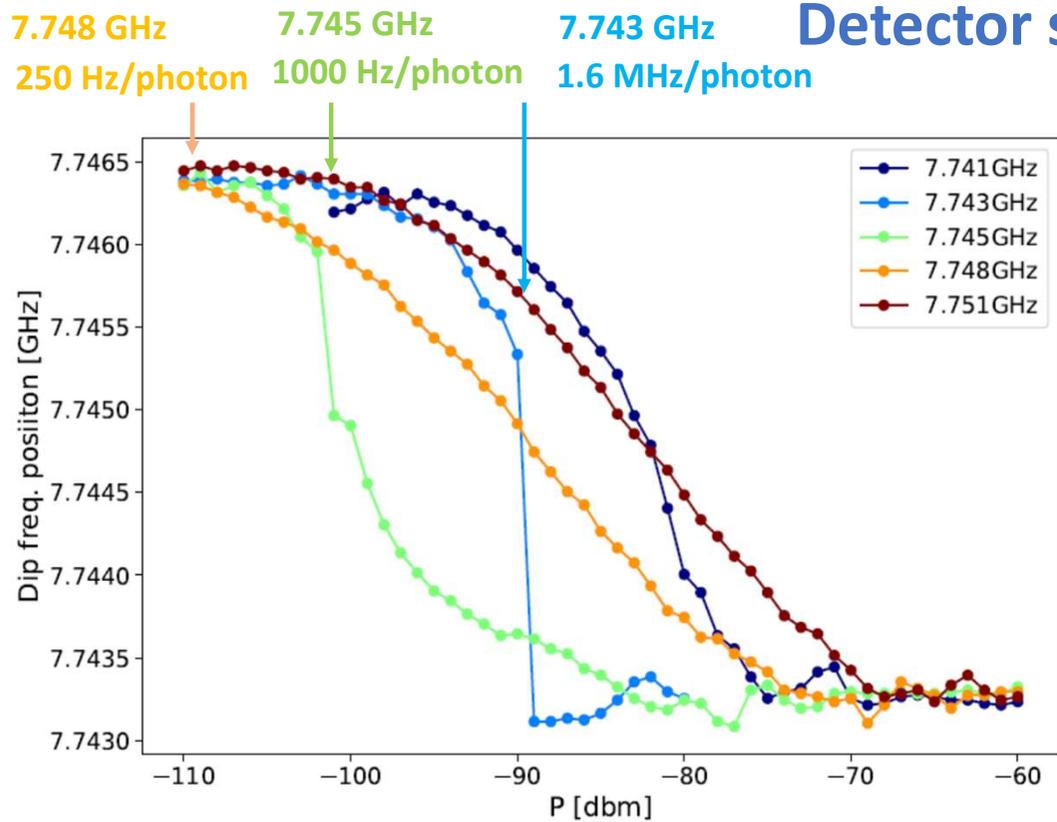
Qubit/qubit Interaction



**Topology-induced
Qubit/Qubit
configurations ...!!**



Detector sensitivity



Biassing the resonator at frequency of 7.743 GHz it could be possible to detect one photon in correspondence of the frequency dip shift of 1.6 MHz, which is an accessible frequency bandwidth included in the working limit of the device.

A) 1990-1999 ELQ, MQT, RMQT ICIB CNR

PHYSICAL REVIEW B

VOLUME 54, NUMBER 2

1 JULY 1996-II

Resonant macroscopic quantum tunneling in SQUID systems

Paolo Silvestrini, Berardo Ruggiero, and Yuri N. Ovchinnikov*
 Istituto di Cibernetica del C.N.R., I-80072, Arco Felice, Napoli, Italy
 (Received 29 January 1996)

Nb

30 mK = T < T_cros

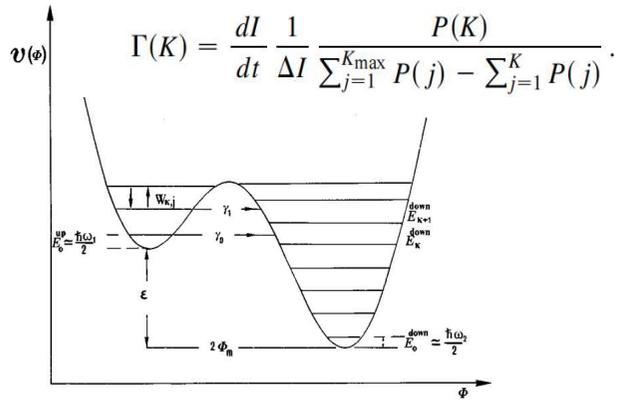


FIG. 1. Sketch of the potential $U(\Phi)$ describing the SQUID dynamics in a quantum picture.

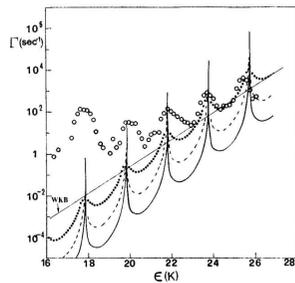
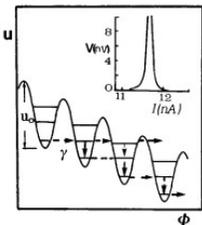
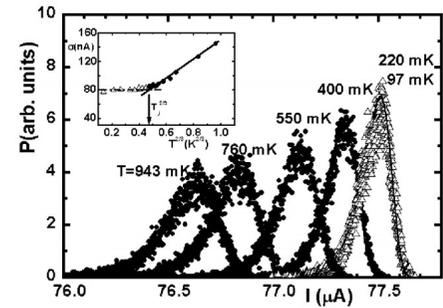
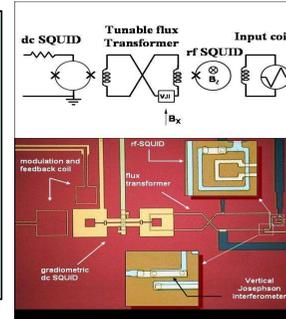
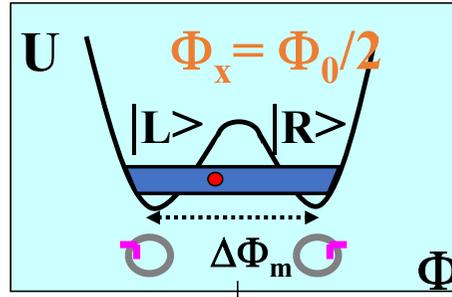


FIG. 2. Escape rate Γ vs ϵ . Open circles are data from Ref. 9, while three different values of resistance have been considered for the theoretical curves: $R/R_Q = 100$ (solid line), $R/R_Q = 10$ (dashed line), $R/R_Q = 1$ (solid points). $R_Q = 6.45$ k Ω . For all the curves $L = 210$ pH, $C = 80$ pF, and $\Delta U_0 = 16.3$ K.

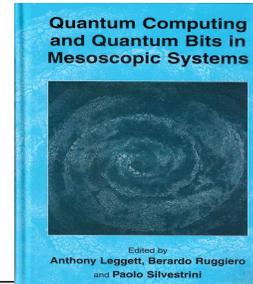
B) 2000-2013 MQC Group ICIB CNR

Nb



Nb rf-SQUID as a qubit

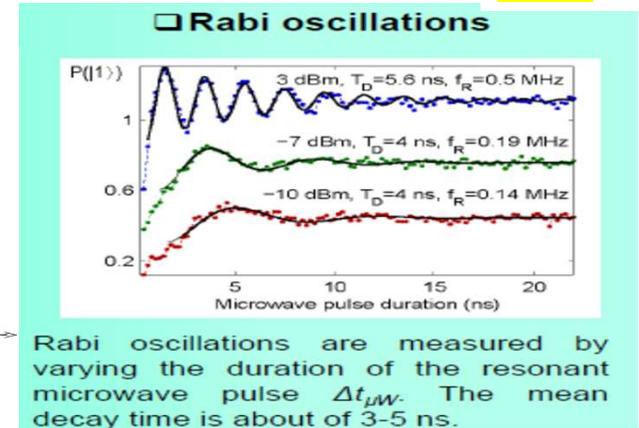
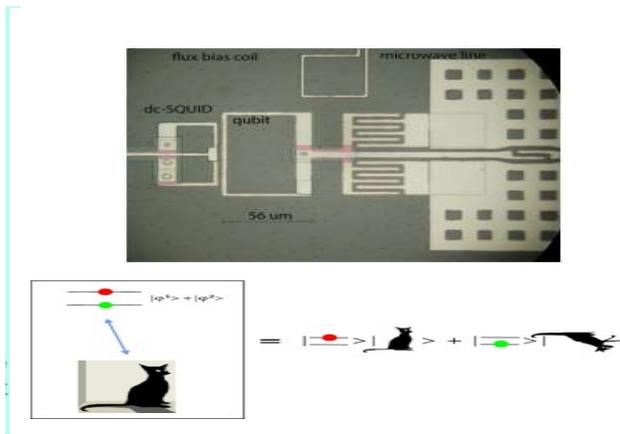
20 mK = T < T_cros



C) 2014-2018 MQC in phase qubits

NbN based superconducting Josephson phase qubit with AlN tunnel barrier
 M.P. Lisitskiy, et al IEEE Trans on Superc. pp.1-3 (2017)

NbN

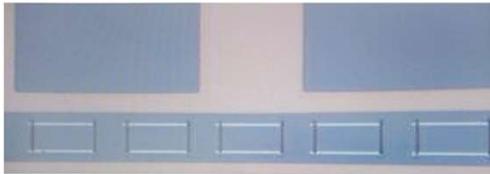


Fabrication a) and Measurements b) (Cross SQN)

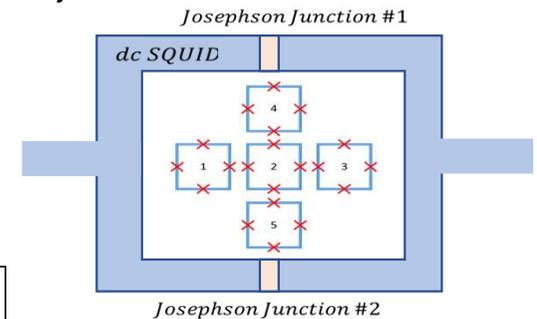
a) EBL ISASI CNR

SQN with 5 flux Qubit with 4 Josephson Junction

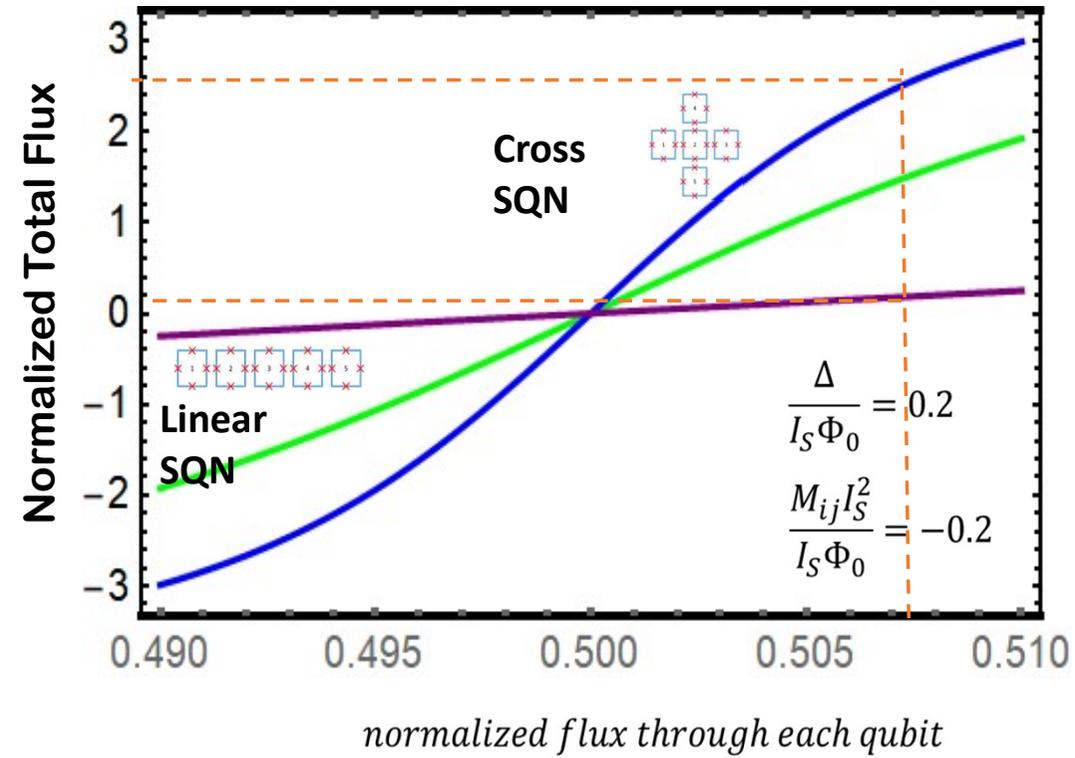
Area JJ = 0.2x0.5 μm^2



Robust QCS in SQN ?



b) ADR Helium Free Cryostat T= 50 mK @ SPIN/ISASI Pozzuoli CNR



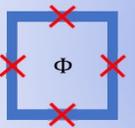
Modeling Superconducting Qubit Networks with 5 flux qubit with inductive coupling

F. Romeo et al. May 2024



a) 4 JJ Flux Qubit Effective Hamiltonian

Qubit effective Hamiltonian



$$\hat{H}_{eff} = -\epsilon(f) \sigma_z - \Delta \sigma_x$$

$$f = \frac{\Phi}{\Phi_0}$$

$$\epsilon(f) = I_S \Phi_0 \left(f - \frac{1}{2} \right)$$

A two-level system

$$R(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{pmatrix}$$

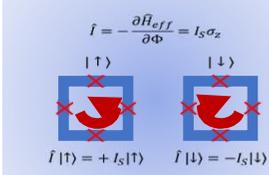
$$\cos(\theta) = \frac{\epsilon}{\sqrt{\epsilon^2 + \Delta^2}}, \quad \sin(\theta) = \frac{\Delta}{\sqrt{\epsilon^2 + \Delta^2}}$$

Coherent superposition of current states,

$$|g; f=0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

$$|e; f=0\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle - |\downarrow\rangle)$$

Current Operator & Current states

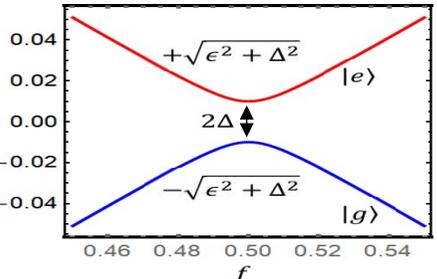
$$\hat{I} = -\frac{\partial \hat{H}_{eff}}{\partial \Phi} = I_S \sigma_x$$


$$\langle \hat{I} \rangle_{|\uparrow\rangle} = +I_S |\uparrow\rangle, \quad \langle \hat{I} \rangle_{|\downarrow\rangle} = -I_S |\downarrow\rangle$$

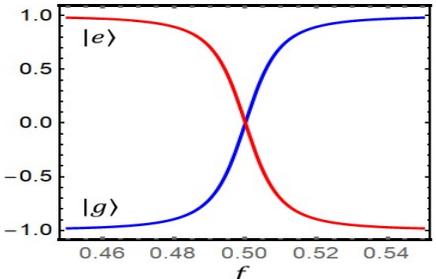
$E_{\pm} = \pm \sqrt{\epsilon^2 + \Delta^2}$

$\langle \hat{I} \rangle_g = \langle g | \hat{I} | g \rangle = +I_S \cos(\theta)$
 $\langle \hat{I} \rangle_e = \langle e | \hat{I} | e \rangle = -I_S \cos(\theta)$

Energy eigenvalues



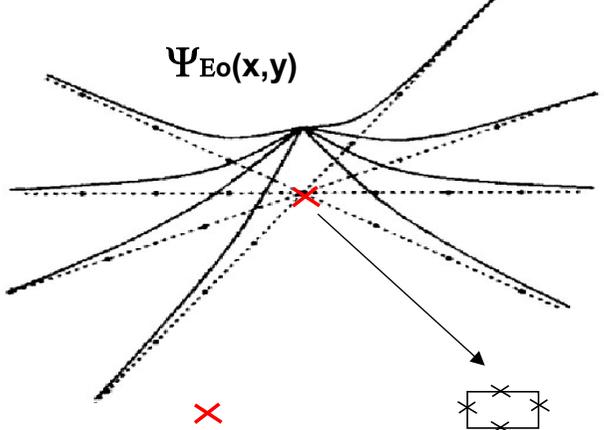
Qubit loop current ⟨Î⟩/Is



$$\cos(\theta) = \frac{\epsilon}{\sqrt{\epsilon^2 + \Delta^2}} = \frac{f - \frac{1}{2}}{\sqrt{(f - \frac{1}{2})^2 + \Gamma^2}}, \quad \Gamma = \frac{\Delta}{I_S \Phi_0}$$

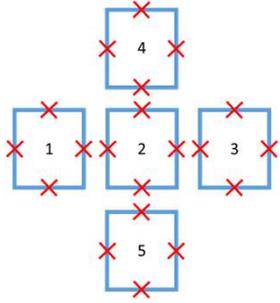
b) SQN topology effects

$\Psi_{E_0}(x,y)$

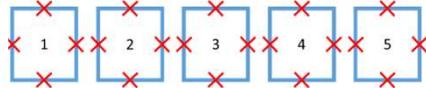


Josephson Junctions → 4 JJ Flux Qubit

Cross SQN



Linear SQN

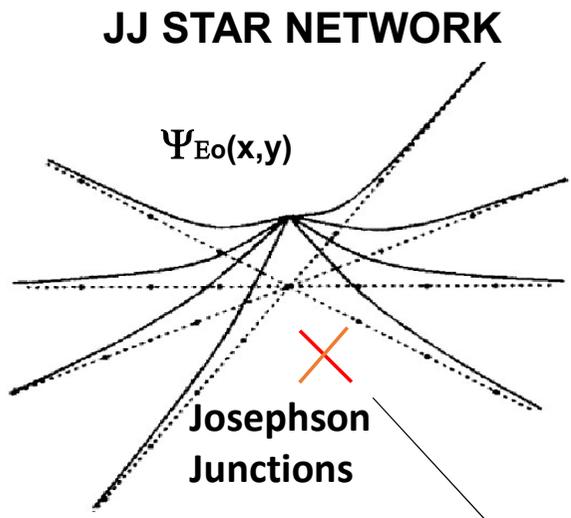


A Many-body Problem

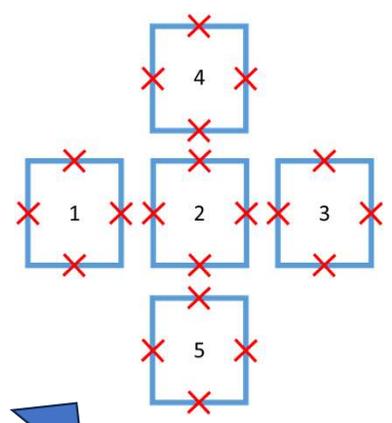
$$H = \sum_i H_q^{(i)} + \frac{1}{2} \sum_{i,j} U_{ij}$$

$$H_q^{(i)} = -[\epsilon_i(f) \sigma_z^{(i)} + \Delta_i \sigma_x^{(i)}]$$

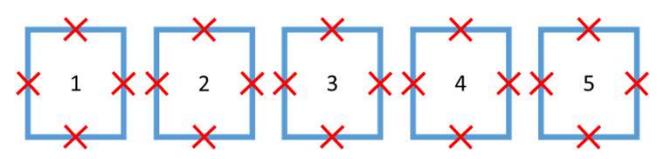
$$U_{ij} = \sum_{i,j} M_{ij} I_S^{(i)} I_S^{(j)} \sigma_z^{(i)} \sigma_z^{(j)}$$



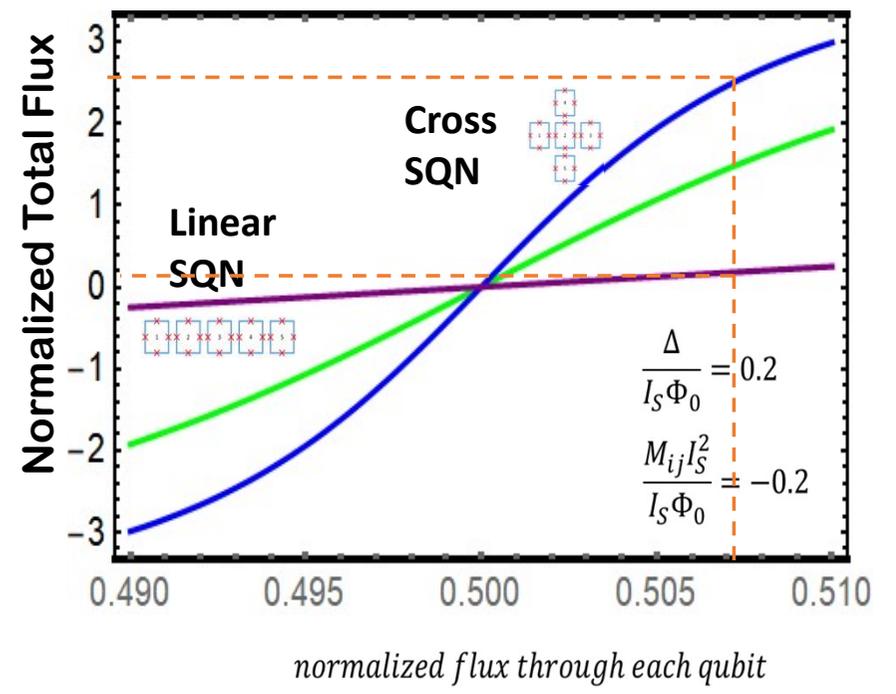
Cross SQN for Quantum Integration



Linear SQN



Robust QCS in SQN ?



Energy eigenvalues (units $I_S \Phi_0$)

